# OPTIMIZING THE KELVIN FORCE IN A SUBDOMAIN

HARBIR ANTIL, RICARDO H. NOCHETTO, AND PABLO VENEGAS

Abstract. In order to approximate the Kelvin (magnetic) force, we propose a minimization problem, with a tracking type cost functional. We use the so-called dipole approximation to realize the magnetic field. Here, the location and the direction of the magnetic source are assumed to be fixed. The magnetic field intensity, with limiting pointwise constraints, acts as the control parameter. We address two specific problems: the first one corresponds to a fixed final time whereas the second one deals with an unknown final time, i.e., is a free boundary problem. We prove the existence of solution and thanks to second order sufficient conditions, we deduce the local uniqueness to these problems under fairly general assumptions on the data. For time discretization we use classical backward Euler scheme. For the first problem we prove the convergence of this semi-discrete numerical scheme using Γ-convergence. We report computational results to assess the validity of the numerical methods. As an application, we study the control of magnetic nanoparticles as those used in magnetic drug delivery.

**Keywords**: Magnetic field design, Kelvin force, non-convex optimization, Γ-convergence.

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George Mason University
*E-mail address*: hantil@gmu.edu

University of Maryland
*E-mail address*: rhn@math.umd.edu

University of Maryland
*E-mail address*: pvenegas@umd.edu