CONSTRUCTION OF HOMOLOGICAL SEIFERT SURFACES AND SOME APPLICATIONS IN ELECTROMAGNETISM

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Abstract. We devise an efficient algorithm for the construction of homological Seifert surfaces in a not simply-connected bounded polyhedral domain $\Omega \subset \mathbb{R}^3$ endowed with a triangulation $\mathcal{T}$. A 1-cycle γ of $\mathcal{T}$ is a formal linear combination (with integer coefficients) of oriented edges of $\mathcal{T}$ with zero boundary. The 1-cycle γ is said to be a 1-boundary of $\mathcal{T}$ if it is equal to the boundary of a formal linear combination S of oriented faces of $\mathcal{T}$. If such a S exists, we call it homological Seifert surface of γ in $\mathcal{T}$. This algorithm allows us to automatically compute a basis of the relative homology group $\mathcal{H}_2(\overline{\Omega}, \partial\Omega; \mathbb{Z})$ that we use in the computation of a discrete approximation of the Hodge decomposition of any field in $(L^2(\Omega))^3$ and in the approximation, using Raviart-Thomas finite elements, of the solution of the curl-div system.

Keywords: homological Seifert surfaces, discrete Hodge decomposition, curl-div system.

Mathematics Subject Classifications (2010): 65D17, 35Q60, 14F40, 52B05.

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