ON SOME DIFFICULTIES OF THE NUMERICAL APPROXIMATION OF NONCONSERVATIVE HYPERBOLIC SYSTEMS

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ABSTRACT. The design of high-order well-balanced shock-capturing numerical methods for nonconservative hyperbolic systems is a very active front of research, as PDE systems of this nature arises in many flow models. The approximated solutions are expected to be consistent with the physics of the real flows to be simulated: in particular (1) they should satisfy the conservation properties prescribed by the physics of the problem and (2) their discontinuities should satisfy some jump conditions consistent with the real phenomena to be simulated.

In this presentation our work [6] will be summarized. The question addressed there is whether or not it is possible to construct numerical schemes satisfying these two requirements. While for conservative systems the answer to this question is positive, some important difficulties arise in certain nonconservative cases. These difficulties will be discussed and illustrated with some numerical results. Finally, some conclusions will be drawn.

Keywords: Nonconservative hyperbolic system, shock waves, family of paths, path-conservative numerical methods, Lax-Wendroff Theorem

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