ADAPTIVE FE–BE COUPLING FOR STRONGLY NONLINEAR TRANSMISSION PROBLEMS WITH CONTACT

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Abstract. The talk is split into two parts. First, we analyze an FE–BE coupling procedure for scalar elastoplastic interface problems involving friction, where a nonlinear uniformly monotone operator such as the p–Laplacian in a bounded Lipschitz domain $\Omega \subset \mathbb{R}^n$ is coupled to the linear Laplace equation on the exterior domain Ω^c. In the second part we present a corresponding coupling formulation for a nonconvex double–well potential in Ω. In both cases the transmission problem is reduced to a boundary/domain variational inequality, which is solved by Galerkin's method with finite and boundary elements. The Galerkin approximations converge in a suitable product of L^p– and L^2–Sobolev spaces.

The nonlinear frictional contact problem under consideration reads for $p \geq 2$: Given $f \in L^{p'}(\Omega)$, $u_0 \in W^{\frac{1}{2},2}(\partial\Omega)$, $t_0 \in W^{-\frac{1}{2},2}(\partial\Omega)$, $g \in L^\infty(\Gamma_s)$ with $\int_\Omega f + \langle t_0, 1\rangle = 0$ for $n = 2$, find minimizers $u_1 \in W^{1,p}(\Omega)$, $u_2 \in W^{1,2}_{loc}(\Omega)$ of the functional

$$(1) \qquad \int_\Omega \varrho(|\nabla u_1|)(\nabla u_1)^2 + \frac{1}{2}\int_{\Omega^c} |\nabla u_2|^2 - \int_\Omega f u_1 - \langle t_0, u_2|_{\partial\Omega}\rangle + \int_{\Gamma_s} g|u_2 - u_1 + u_0|,$$

$\partial\Omega = \overline{\Gamma_s \cup \Gamma_t}$, over a convex subset of $W^{1,p}(\Omega) \times W^{1,2}_{loc}(\Omega)$ encoding the transmission condition on Γ_t. Here $\varrho(t)$ is a function $\varrho(x,t) \in C(\overline{\Omega} \times (0,\infty))$ satisfying

$$0 \leq \varrho(t) \leq \varrho^*[t^\delta(1+t)^{1-\delta}]^{p-2}, \ |\varrho(t)t - \varrho(s)s| \leq \varrho^*[(t+s)^\delta(1+t+s)^{1-\delta}]^{p-2}|t-s|,$$

and $\varrho(t)t - \varrho(s)s \geq \varrho_*[(t+s)^\delta(1+t+s)^{1-\delta}]^{p-2}(t-s)$ for all $t \geq s > 0$ uniformly in $x \in \Omega$ ($\delta \in [0,1]$, $\varrho_*, \varrho^* > 0$).

To reduce the exterior problem to $\partial\Omega = \partial\Omega^c$, we use the Steklov–Poincaré operator $S : W^{\frac{1}{2},2}(\partial\Omega) \to W^{-\frac{1}{2},2}(\partial\Omega)$ for the Laplacian on Ω^c. The problem translates into a domain/boundary variational inequality: Find $(\hat{u},\hat{v}) \in X$ such that for all $(u,v) \in X = W^{1,p}(\Omega) \times \{v \in W^{\frac{1}{2},2}(\partial\Omega) : \text{supp } v \subset \Gamma_s\}$,

$$\int_\Omega \varrho(|\nabla\hat{u}|)\nabla\hat{u}\nabla(u-\hat{u}) + \langle S(\hat{u}|_{\partial\Omega} + \hat{v}), (u-\hat{u})|_{\partial\Omega} + v - \hat{v}\rangle + \int_{\Gamma_s} g(|v| - |\hat{v}|) \geq \lambda(u-\hat{u}, v-\hat{v}).$$

Theorem 1. *The variational inequality is equivalent to the minimization problem (1) and has a unique solution.*

For a family of finite dimensional subspaces $X_h = H^1_h \times H^{\frac{1}{2}}_h$ of X, $h \in I$, we present a priori error estimates.

Remark 1. *The above procedure carries over to transmission problems in nonlinear elasticity with a Hencky material in Ω and the Lamé equation in Ω^c.*

Next, we consider an FE–BE coupling for transmission problems with microstructure and Signorini contact. Our starting point is the relaxed energy functional

$$\Phi^{**}(u_1,u_2) = \int_\Omega W^{**}(\nabla u_1) + \frac{1}{2}\int_{\Omega^c}|\nabla u_2|^2 - \int_\Omega f u_1 - \langle t_0, u_2|_{\partial\Omega}\rangle,$$

where W^{**} is the convex envelope of the double–well potential $W(F) = |F - F_1|^2 \, |F - F_2|^2$ for $F_1 \neq F_2 \in \mathbb{R}^n$. The minimization problem for Φ^{**} corresponds to the variational inequality: Find $(\hat{u},\hat{v}) \in \mathcal{A} = \{(u,v) \in W^{1,4}(\Omega) \times W^{\frac{1}{2},2}(\partial\Omega) : v|_{\Gamma_s} \geq 0, \ \langle S(u|_{\partial\Omega} + v - u_0), 1\rangle = 0 \text{ if } n = 2\}$ such that

$$\int_\Omega DW^{**}(\nabla\hat{u})\nabla(u-\hat{u}) + \langle S(\hat{u}|_{\partial\Omega} + \hat{v}), (u-\hat{u})|_{\partial\Omega} + v - \hat{v}\rangle \geq \lambda(u-\hat{u}, v-\hat{v})$$

for all $(u,v) \in \mathcal{A}$. We show that the stress $DW^{**}(\hat{u})$, a certain projection $\mathbb{P}\nabla\hat{u}$ of the gradient, the region of microstructure and the boundary value $u|_{\partial\Omega} + v$ are independent of the minimizer and present a priori error estimates for the FE–BE approximation.

[1] H. Gimperlein, M. Maischak, E. Schrohe, E. P. Stephan. Adaptive FE–BE coupling for strongly nonlinear transmission problems with Coulomb friction. Preprint, 2009.

[2] H. Gimperlein, E. Schrohe, E. P. Stephan. FE–BE coupling for a transmission problem involving microstructure. In preparation, 2009.

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