# ERROR ESTIMATES FOR THE FINITE VOLUME METHOD FOR A COPPER HEAP LEACHING MODEL

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Abstract. This work is motivated by a combined mixed finite element (MFE) - finite volume (FV) scheme of a two phase flow model for the heap leaching of copper ores modeled by a degenerate parabolic equation

$$\partial_t u - \nabla \cdot (\nabla \beta(u) + F(u)) = r(u), \quad \text{in } Q_T \equiv (0,T) \times \Omega.$$

Initially we have $u(0) = u^0$ in $\Omega$, whereas $u = 0$ on $\partial\Omega$. In the above $0 < T < \infty$ is fixed, $\Omega$ is a bounded domain in $\mathbb{R}^d (d \geq 1)$ with a Lipschitz continuous boundary. The function $\beta : \mathbb{R} \to \mathbb{R}$ is non-decreasing and differentiable. By degeneracy we mean a vanishing diffusion, namely $\beta'(u) = 0$ for some $u$. We prove error estimates for the finite volume discretization for this model. Several numerical results illustrating the performance of the algorithm are provided.

**Keywords**: Finite Volume Methods, error estimates, porous medium, heap leaching, copper ores.

## References

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