MATHEMATICAL AND NUMERICAL MODELING OF PIEZOELECTRIC SENSORS

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Abstract. Piezoelectric sensors are widely used for ultrasonic non destructive testing as they can convert an elastodynamic wave into a difference of potential and vice versa, they are used both as emmiter and receiver. The behaviour of such devices is governed by the equations of piezoelectricity. Piezoelectric bars have been studied decades ago ([1],[2]), these models are either too simple (1D Approximation) or in frequential domain, and assume a very simple geometry. Our goal is to present a mathematical model in time domain for general piezoelectric sensors using the full mathematical model.

The equations of piezoelectricity result from a coupling with linear Maxwell equations and linear elastodynamic equations : see ([3]) for the mathematical analysis of this system. The unkowns of the problem are both the displacement field and the electromagnetic field. To simplify the model, we use the well known electrostatic approximation which reduces the electromagnetic part of the unkowns to a scalar potential φ. We justify mathematically this approach by a limit process considering the inverse of the speed of light as a small paramater. The full system to be solved in time domain couples the classical elastodynamic equations on the solid domain (Ω_S) coupled with a Laplace equation posed a priori in the whole domain.

$$\rho \partial_{tt} u - \mathbf{div}\ C\varepsilon(u) - \mathbf{div}\ \mathbf{e}\nabla\varphi = 0 \text{ in } \Omega_S,$$

$$\nabla \cdot \epsilon \nabla \varphi - \nabla \cdot \mathbf{e}^T \varepsilon(u) = 0 \text{ in } \mathbb{R}^3.$$

ρ is the density, C is the classical elastic tensor and ϵ is the permittivity. $\mathbf{e}$ is the non standard piezoelectric tensor that couples both equations. Considering the large contrast of permittivity between various materials, we restrict the domain of computation of φ to a subdomain of Ω_S. This is also justify by a limit process : small permittivities are considered as small parameters.

We wish to model both emission and reception processes. This will induce particular boundary conditions. In the emission process, a potential is applied to both side of the piezoelectric materials, this corresponds to a simple Dirichlet condition that closes the Laplace equation. During the reception process, both sides of the piezoelectric materials are connected to a resistor, the current being directly proportional to the voltage we obtain on one of the boundary a relation of the form :

$$\varphi = \partial_t \int_\Gamma \nabla\varphi \cdot \mathbf{n}.$$

We shall present a numerical method for handling the problem combining high order galerkin finite elements ([4]) and explicit time discretization. Numerical results will be presented.

Keywords: piezoelectricity, modeling, asymptotic analysis, high order finite elements

Mathematics Subject Classifications (2000): 35J25,35Q61,35J05,35B27

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