V A L P A R A Í S O N U M É R I C O IV

Séptimo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso
Valparaíso, Chile, Diciembre 11 - 13, 2013

PROGRAMA y RESUMENES

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1. Introducción

El **Séptimo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales** ha sido organizado en conferencias secuenciales de **45 y 30 minutos** de duración (40 y 25 minutos de exposición, respectivamente, y 5 minutos para preguntas y comentarios). Todas las charlas se llevarán a cabo en el Auditorio de la Facultad de Ingeniería de la **Pontificia Universidad Católica de Valparaíso**.

En las páginas siguientes se detalla la programación correspondiente. Cuando hay más de un autor, aquel que aparece subrayado corresponde al expositor.

Los organizadores expresamos nuestro agradecimiento a los auspiciadores que se indican a continuación, los cuales han aportado gran parte de los recursos necesarios para el financiamiento de este evento:

- Conicyt, Gobierno de Chile, a través del Anillo ACT 1118 (ANANUM), del Centro de Investigación en Ingeniería Matemática (CI^{2}MA) de la Universidad de Concepción,
- Pontificia Universidad Católica de Valparaíso,
- Centro de Modelamiento Matemático (CMM) de la Universidad de Chile, y
- Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

Igualmente, extendemos nuestro reconocimiento y gratitud a todos los expositores, quienes han hecho posible la realización de **Valparaíso Numérico IV**.

Comité Organizador

Valparaíso, Diciembre 2013

2. Miércoles, 11 de Diciembre

8.30-9.15 INSCRIPCIÓN

9.15-9.30 PALABRAS DE BIENVENIDA

[*Moderador*: I. MUGA]

9.30-10.15 Daniele Boffi: *The finite element immersed boundary method for fluid structure interactions: A fictitious domain approach.*

10.15-10.45 Lothar Banz, Heiko Gimplerlein, Abderrahman Issaoui, Ernst P. Stephan: *Stabilized hp-BEM for frictional contact problem in linear elasticity.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Norbert Heuer: *On the equivalence of fractional-order Sobolev semi-norms.*

11.45-12.15 Johnny Guzmán, Manuel Sánchez-Uribe: *Max-norm stability of low order Taylor-Hood elements in three dimensions.*

12.15-12.45 Raimund Bürger, Ricardo Ruiz-Baier, Canrong Tian: *Stability analysis and finite volume element discretization for delay-driven spatial patterns in a predator-prey model.*

12.45-15.00 ALMUERZO

[*Moderador*: N. HEUER]

15.00-15.45 L. Beirão da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L.D. Marini, A. Russo: *An introduction to the virtual element method.*

15.45-16.15 Juan Carlos de los Reyes, Sergio González-Andrade: *Numerical simulation of thermally convective viscoplastic fluids by semismooth second order type methods.*

16.15-16.45 Norbert Heuer, Michael Karkulik: *DPG boundary elements with optimal test functions on surfaces.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Andrés I. Ávila, Andreas Meister, Martin Steigemann: *On numerical methods for nonlinear singularly perturbed Schrödinger problems.*

17.45-18.15 María G. Armentano: *A posteriori error estimates for an hp finite element method.*

18.15-18.45 Gabriel N. Gatica, George C. Hsiao, Salim Meddahi, Francisco-Javier Sayas: *New developments on the coupling of mixed-FEM and BEM for the three-dimensional Stokes problem.*

19.30 COCKTAIL DE BIENVENIDA

3. Jueves, 12 de Diciembre

[*Moderador*: G. GATICA]

9.30-10.15 Helmut Harbrecht: *Analytical and numerical methods in shape optimization.*

10.15-10.45 Eduardo Lara, Rodolfo Rodríguez, Pablo Venegas: *Finite element approximation of the eigenvalue problem for the* **curl** *operator in multiply connected domains.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Ana Alonso R., Jessika Camaño, Rodolfo Rodríguez, Alberto Valli: *Comparison of two computational models for the inverse problem of electroencephalography.*

11.45-12.15 Carlos Jerez-Hanckes: *Local multiple traces formulation: theoretical extensions and novel applications.*

12.15-12.45 Manuel Solano, Bernardo Cockburn: *On the robustness of a hybridizable discontinuous Galerkin method for curved domains.*

12.45-15.00 ALMUERZO

[*Moderador*: R. RODRÍGUEZ]

15.00-15.45 Carsten Carstensen, Michael Feischl, Dirk Praetorius: *Rate optimality of adaptive algorithms: An axiomatic approach.*

15.45-16.15 Leonardo E. Figueroa: *Greedy-type algorithms based on finite element discretizations approximating elliptic PDE on cartesian product domains.*

16.15-16.45 Juan Carlos de los Reyes, Estefana Loayza, Pedro Merino: *On the use of second order information for the numerical solution of PDE-constrained optimization problems with sparsity.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Alfredo Bermúdez, M. Dolores Gómez, Rodolfo Rodríguez, Pablo Venegas: *Computational assessment of a finite element method for axisymmetric eddy current problems with hysteresis.*

17.45-18.15 Ricardo Ruiz-Baier, Héctor Torres: *Numerical solution of a multidimensional sedimentation problem using finite volume-element methods.*

18.15-18.45 Ricardo Oyarzúa: *A conforming mixed finite element method for the Navier-Stokes/Darcy coupled problem.*

20.30 CENA DE CAMARADERÍA

4. Viernes, 13 de Diciembre

[*Moderador*: R. BÜRGER]

9.30-10.15 Siddhartha Mishra: *Numerical approximation of entropy measure valued solutions of hyperbolic conservation laws.*

10.15-10.45 Verónica Anaya, Mostafa Bendahmane, Mauricio Sepúlveda: *Convergent finite volume schemes for nonlocal and cross diffusion reaction equations. Applications to biology.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Fernando A. Morales: *Well-posedness and convergence of a primal-dual hybrid discontinuous Galerkin scheme for porous media in 2-D.*

11.45-12.15 Mauricio A. Barrientos, Matthias Maischak: *A dual-mixed analysis for incompressible quasi-Newtonian flows.*

12.15-12.45 Salim Meddahi, David Mora, Rodolfo Rodríguez: *A finite element analysis of a pseudostress formulation for the Stokes eigenvalue problem.*

12.45-15.00 ALMUERZO

[*Moderador*: R. OYARZÚA]

15.00-15.45 Hélène Barucq, Henri Calandra, Julien Diaz, Florent Ventimiglia: *On the use of high-order schemes for seismic imaging.*

15.45-16.15 Sergio Caucao, David Mora, Ricardo Oyarzúa: *Analysis of a mixed finite element method for the Stokes problem with varying density in pseudostress-velocity formulation.*

16.15-16.45 Luis M. Villada, Raimund Bürger, Christophe Chalons: *Lagrangian-remap schemes for multi-species kinematic flow problems.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Erwan Hingant, Mauricio Sepúlveda: *Numerical approximation of a sorption-coagulation equation.*

17.45-18.15 Jay Gopalakrishnan, Ignacio Muga, Nicole Olivares: *Dispersive and dissipative errors in the DPG method with scaled norms for Helmholtz equation.*

5. Resúmenes

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Convergent finite volume schemes for nonlocal and cross diffusion reaction equations. Applications to biology.*

Verónica Anaya[†], Mostafa Bendahmane[‡], Mauricio Sepúlveda[§]

Abstract

In this work, we consider reaction-diffusion systems with nonlocal and cross diffusion. We construct a finite volume scheme for this system, we establish existence and uniqueness of the discrete solution, and it is also showed that the scheme converges to the corresponding weak solution for the model studied. The convergence proof is based on the use of the discrete Sobolev embedding inequalities with general boundary conditions and a space-time L^1 compactness argument that mimics the compactness lemma due to S. N. Kruzhkov. The first example of application is the description of three interacting species in a HP food chain structure. The second example of application corresponds to a mathematical model with cross-diffusion for the indirect transmission between two spatially distributed host populations having non-coincident spatial domains, transmission occurring through a contaminated environment. We give also, several numerical examples.

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[†]Departamento de Matemáticas, Universidad del Bío-Bío, Chile, e-mail: vanaya@ubiobio.cl

[‡]Institut Mathematiques, Universite Victor Segalen Bordeaux 2, France, e-mail: mostafa.bendahmane@u-bordeaux2.fr

[§]**CI^{2}MA** and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: msepulveda@ci2ma.udec.cl

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Comparison of two computational models for the inverse problem of electroencephalography.*

Ana Alonso R.[†], Jessika Camaño[‡], Rodolfo Rodríguez[§], Alberto Valli[¶]

Abstract

The goal of this work is to assess two approximation methods for the inverse problem of electroencephalography: the localization of brain activity from measurements of the electric potential on the surface of the head. The source current is modeled as a dipole whose localization and polarization has to be determined. The two considered methods are the so called *subtraction approach* and *direct approach*. The former is based on subtracting a fundamental solution, which has the same singular character of the actual solution, and solving computationally the resulting non-singular problem. Instead, the latter consists in solving directly the problem with singular data by means of an adaptive process based on an a posteriori error estimator, which allows creating meshes appropriately refined around the singularity. A set of experimental tests for both, the forward and the inverse problem, are reported. The main conclusion of these tests is that the direct approach combined with adaptivity is preferable when the localization of the dipole is close to an interface.

Key words: inverse problem, dipole source, electrostatic, electroencephalography

Mathematics subject classifications: 65N15, 65N21, 65N30

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[†]Department of Mathematics, University of Trento, Italy, e-mail: alonso@science.unitn.it

[‡]**CI^{2}MA** and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: jcamano@ing-mat.udec.cl

[§]**CI^{2}MA** and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: rodolfo@ing-mat.udec.cl

[¶]Department of Mathematics, University of Trento, Italy, e-mail: valli@science.unitn.it

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A posteriori error estimates for an *hp* finite element method.*

María G. Armentano†

Abstract

In this talk we first introduce an *hp* finite element method to solve two-dimensional fluid-structure spectral problems in polygonal domains, which arise from the computation of the vibration modes of a bundle of parallel tubes immersed in an incompressible fluid [1, 2, 4]. We prove the convergence of the method and we define an a posteriori error estimator of residual type which can be computed locally from the approximate eigenpair. We show its reliability and efficiency by proving that the estimator is equivalent to the energy norm of the error up to higher order terms, the equivalence constant of the efficiency estimate being suboptimal in the sense that it depends on the polynomial degree. Following the *hp* adaptive strategy given in [5] we present an *hp* adaptive algorithm and several numerical tests which show the performance of the scheme, including some numerical evidence of exponential convergence. Then, we also present an *hp* finite element adaptive scheme to solve a source problem on curved domains. We show the advantage of using curved triangles [6] and we exhibit the loss of convergence rate if we use standard triangular elements with straight edges [3].

Key words: a posteriori error estimates, *hp* finite elements method.

Mathematics subject classifications (1991): 65N30, 65N25, 65N15

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†IMAS-CONICET and Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, CP 1428, Buenos Aires, Argentina , e-mail: garmenta@dm.uba.ar

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On numerical methods for nonlinear singularly perturbed Schrödinger problems.

Andrés I. Ávila*, Andreas Meister†, Martin Steigemann‡

Abstract

Nonlinear Schrödinger equations (NSE) model several important problems in Quantum Physics and Morphogenesis. In case of singularly perturbed problems, the theory have made interesting progress, but numerical methods have been not able to come up with small values of the singular parameter ε. Moreover, the saddle-point characteristic of the associated functional is another challenge that it was first studied by Choi & McKenna, who developed the Mountain Pass Algorithm. We will focus on NSE where a uniqueness result for ground-state solutions is obtained. In this article, we develop a new method which improves the results for a large range of singular parameters. We extend the MPA ideas considering the singulary perturbed problems by developing a finite element approach mixed with steepest descend directions. We use a modified line search method based on Armijo's rule for improving the Newton search and Patankar trick for preserving the positiveness of the solution. To improve the range of the singular parameter, adaptive methods based on Dual Weighted Residual method are used. Our numerical experiments are performed with the deal.II library and we show that it is possible to get solutions for $\varepsilon = 10^{-6}$ improving the current results in four order of magnitude. At this level, machine precision must be considered for further studies.

Keywords: singularly perturbed Schrödinger problems, Mountain Pass Algorithm, Patankar trick, dual weighted residual method

Mathematics subject classifications (1991): 35Q55, 65N30, 65N50

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*Centro de Modelación y Computación Científica, Universidad de La Frontera, Temuco, Chile, e-mail: andres.avila@ufrontera.cl

†Department of Mathematics and Natural Sciences, Institute of Mathematics, University of Kassel, Germany, e-mail: meister@mathematik.uni-kassel.de

‡Department of Mathematics and Natural Sciences, Institute of Mathematics, University of Kassel, Germany, e-mail: martin.steigemann@mathematik.uni-kassel.d

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Stabilized *hp*-BEM for frictional contact problem in linear elasticity.

Lothar Banz*, Heiko Gimplerlein†, Abderrahman Issaoui*, Ernst P. Stephan*

Abstract

We consider Tresca-frictional contact for linear elasticity in $\mathcal{R}^2$. We use the Poincare-Steklov operator, which realizes the Dirichlet-to-Neumann map, and represent the negative of the unknown traction on the contact boundary by a vector-valued Lagrange multiplier. In order to avoid the discrete inf-sup-condition a mesh-dependent stabilization term is added to the discrete mixed formulation. In particular, this allows the use of the same mesh on the contact boundary for both primal and dual variables, which, from an implentation point of view, is desireable. The resulting discrete formulation can be solved efficiently by a semi-smooth Newton algorithm. We prove the well-posedness of the formulation as well as an a priori and a posteriori error estimate. Numerical experiments are given which support our theoretical results.

Key words: Stabilized mixed boundary elements, Tresca friction, Signorini contact

Mathematics subject classifications (1991): 65N38, 65N12, 65N15

*Institute of Applied Mathematics, Leibniz University Hannover, Hannover, Germany, e-mail: {banz,issaoui,stephan} @ifam.uni-hannover.de

†School of Mathematical & Computer Sciences, Mathematics, Heriot-Watt University, Edinburgh, Scotland, e-mail: H.Gimperlein@hw.ac.uk

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A dual-mixed analysis for incompressible quasi-Newtonian flows.

Mauricio A. Barrientos*, Matthias Maischak†

Abstract

We consider the coupling of dual-mixed finite element method and boundary integral equation method to solve a transmission problem between a lineal Stokes flow with a quasi-Newtonian flow with mixed boundary conditions. The result is a new mixed scheme for the quasi-Newtonian problem. The approach is based on the introduction of both the flux and the strain tensor as further unknowns, which yields a two-fold saddle point operator equation as the resulting variational formulation. We derive existence and uniqueness of solution for the continuous and discrete formulations and provide the associated error analysis. In particular, the corresponding Galerkin scheme is defined by using piecewise constant functions and Raviart-Thomas spaces of lowest order. Most of our analysis makes use of an extension of the classical Babuska-Brezzi theory to a class of nonlinear saddle-point problems. Also, we develop a-posteriori error estimates (based on Bank-Weiser type) and propose and reliable adaptive algorithm to compute the finite elements solutions. Finally, several numerical results are provided.

Key words: mixed finite elements, a-posterior error estimates

Mathematics subject classifications (1991): 65N30, 65N15, 76D07, 76M10.

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*Instituto de Matemáticas, Facultad de Ciencias, Pontificia Universidad Católica de Valaparaíso,Valaparaíso Chile, e-mail: mauricio.barrientos@ucv.cl

†BICOM, Brunel University, Uxbridge, UB8 3PH, UK., e-mail: matthias.maischak@brunel.ac.uk

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On the use of high-order schemes for seismic imaging.*

Hélène Barucq†, Henri Calandra‡, Julien Diaz†,
Florent Ventimiglia†

Abstract

Seismic imaging involves the propagation of waves generated by artificial sources to produce maps of the subsurface. Images are reconstructed from measurements by sensors recording signals from the reflected waves which contain two types of information. The first one contains the kinematics of the propagation phenomenon and its determination is crucial to pinpoint the different reflectors. Then, computing the kinematics accurately provides an efficient tool capable of drawing the edges of each structural region composing the subsurface. The second one reproduces the dynamics of the propagation medium which is used to determine the material properties of each geological layer constituting the subsurface. From a mathematical point of view, both issues belong to the class of inverse problems but they do not use the solutions of wave equations in the same way. This work focuses on numerical methods which are used to deliver information on the kinematics related to the propagation of waves in heterogeneous media. Applied mathematicians use to concentrate efforts on the accuracy of the numerical solutions by developing more and more advanced numerical methods. But, in the context of seismic imaging, this is not the only drawback. Indeed, seismic imaging delivers images of the subsurface from the cross-correlation of a collection of solutions of wave equations. Then, for a given accuracy, conventional numerical methods quickly reach their limitations because the production of images require to store away a huge number of snapshots. Hence, an advanced numerical method for seismic imaging must deliver accurate solutions to wave equations with an optimal use of the computer memory. Numerical methods such as finite element schemes are well-known to capture accurately the properties of wave propagation in highly heterogeneous media. The price to pay is high computational burdens which can be partially balanced by the use of parallel computing. Numerical methods for seismic imaging must thus involve computations which can be well-distributed to the processors. Following this point of view, Discontinuous Galerkin (DG) approximations are very attractive since they are suitable for parallel computations and provide very flexible approximation tools particularly adapted to reproduce wave propagation in highly contrasted media. DG methods are moreover *hp*-adaptive and can be applied on general meshes composed of tetrahedra or hexahedra. They are therefore able to limit the occupation of memory as they allow to optimize

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†**Inria** and Laboratory of Applied Mathematics at Pau, University of Pau, France, e-mails: helene.barucq@inria.fr, julien.diaz@inria.fr, florent.ventimiglia@inria.fr

‡CSTJF, Total, Pau, France, e-mail: henri.calandra@total.fr

the number of degrees of freedom due to the combination of different orders of approximation determined by regional physical characteristics of the propagation medium. DG methods have also the noteworthy advantage to produce block-diagonal mass matrices. The discretization procedure can thus be completed with an explicit time approximation which increases the capability of the numerical method to stay within the limits of the memory space. The Leap-Frog scheme is the most widely used explicit time discretization. It is of order 2 only and therefore, it does not let to take fully advantage of high order space discretization. High-order time schemes have then been developed and DG-ADER methods [2] emerged as extension of the Modified Equation Technique [3, 4]. Regarding the memory use, DG-ADER schemes are relevant because they are single step time integration procedures. Thus, they only require to store the solution at the previous time step. Nevertheless, memory limits can be reached in particular when solving 3D wave equations because they require to introduce auxiliary unknowns. A time scheme which requires less memory than DG-ADER methods for a given level of accuracy is thus mandatory for seismic imaging. The purpose of this work is to construct a new higher order time scheme for wave equations by exploiting the capability of DG functions to easily approximate high order differential operators. A Modified Equation Technique can thus be applied, but by applying the time integration first. High order differential operators are then introduced but their discretization by a DG method is straightforward. The resulting single-step time scheme is of arbitrary order and demonstrates a high level of accuracy while creating acceptable computational costs. In particular, for a given accuracy, the new scheme allows for using coarser meshes than with DG-ADER methods. The storage and the computational times are thus considerably reduced. These concluding remarks are obtained by validating the time scheme with the DG formulation proposed in [1] on toy problems. The validation for realistic configurations for seismic imaging sets next the difficult question of introducing seismic sources. This issue suggests to modify the DG scheme proposed in [1] by adding a penalization term which may hamper the performances of explicit time discretization schemes. We illustrate this point by performing numerical experiments in realistic configurations.

Key words: Discontinuous Galerkin approximations, higher order explicit time schemes, elastodynamic equation, seismic imaging

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 35J05

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

An introduction to the virtual element method.

L. Beirão da Veiga [*], F. Brezzi [†], A. Cangiani [‡],
G. Manzini [§], L.D. Marini [¶], A. Russo [‖]

Abstract

The Virtual Element Method (VEM) is a very recent technology [1] for the discretization of partial differential equations. The VEM can be interpreted as a novel approach that shares the same variational background as the Finite Element Method but enjoys also a connection with modern Mimetic schemes. By avoiding the explicit integration of the shape functions that span the discrete Galerkin space and introducing a novel construction of the associated stiffness matrixes, the VEM acquires very interesting properties and advantages with respect to more standard Galerkin methods, yet still keeping the same coding complexity. For instance, the VEM easily allows for polygonal/polyhedral meshes (even non-conforming) with non-convex elements and possibly with curved faces; it allows for discrete solutions of arbitrary C^k regularity, defined on unstructured meshes. The present talk is an introduction to the VEM, aiming at showing the main ideas of the method from [1]. After introducing the method on a simple model problem, we will present an (optimal) convergence result and some numerical tests. We will moreover address the practical construction of the scheme [1, 6] and also some interesting possibilities such as that of using high regularity discrete spaces [2]. Other recent advances on the VEM, here not described, are [3, 4, 5].

Key words: Galerkin approximation, polygonal and polyhedral meshes, Virtual Element Method

Mathematics subject classifications (2010): 65N30, 65N99.

[*]Dep. of Mathematics, University of Milan, e-mail: lourenco.beirao@iunimi.it
[†]IUSS, Pavia (Italy), e-mail: brezzi@imati.cnr.it
[‡]Dep. of Mathematics, University of Leicester, e-mail: andrea.cangiani@le.ac.uk
[§]LANL, Los Alamos, and IMATI-CNR, Pavia, e-mail: manzini@imati.cnr.it.
[¶]Dep. of Mathematics, University of Pavia, e-mail: marini@imati.cnr.it.
[‖]Dep. of Math. and Appl., University of Milan-Bicocca, e-mail alessandro.russo@unimib.it.

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Computational assessment of a finite element method for axisymmetric eddy current problems with hysteresis. *

Alfredo Bermúdez†, M. Dolores Gómez†, Rodolfo Rodríguez‡, Pablo Venegas‡

Abstract

This work deals with the mathematical analysis and the computation of transient electromagnetic fields in nonlinear magnetic media with hysteresis. The results obtained complement those in [1, 2], where the mathematical and numerical analysis of a 2D nonlinear axisymmetric eddy current model was performed under fairly general assumptions on the **H**–**B** curve but without considering hysteresis effects. In our case, the constitutive relation between **H** and **B** is given by a hysteresis operator, i.e., the values of the magnetic induction depend not only on the present values of the magnetic field but also on its past history. Like in [1], we assume axisymmetry of the fields and then we consider two kinds of boundary conditions. Firstly the magnetic field is given on the boundary (Dirichlet boundary condition). Secondly, the magnetic flux through a meridional plane is given, leading to a non-standard boundary-value problem. For both problems, an existence result is achieved under suitable assumptions. For the numerical solution, we consider the Preisach model as hysteresis operator, a finite element discretization by piecewise linear functions, and the backward Euler time-discretization. We report a numerical test in order to assess the order of convergence of the proposed numerical method. Finally, we validate the numerical scheme with experimental results. With this aim, we consider a physical application: the numerical computation of eddy current losses in laminated media as those used in transformers or electric machines.

Key words: transient eddy current, axisymmetric problem, hysteresis, finite elements

Mathematics subject classifications (1991): 65M60, 78A55, 78M10

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†Departamento de Matemática Aplicada, Universidad de Santiago de Compostela, Campus Sur s/n, E-15782, Santiago de Compostela, España, e-mail: alfredo.bermudez@usc.es, mdolores.gomez@usc.es

‡**CI^{2}MA** and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: rodolfo@ing-mat.udec.cl, pvenegas@ing-mat.udec.cl

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

The finite element immersed boundary method for fluid structure interactions: A fictitious domain approach.

Daniele Boffi*

Abstract

The Immersed Boundary Method (IBM) has been introduced by Peskin in the 70's in order to model and approximate fluid-structure interaction problems related to the blood flow in the heart. The original scheme makes use of finite differences for the discretization of the Navier–Stokes equations. We introduced a finite element formulation which has the advantage of handling the presence of the solid (modeled via a Dirac delta function) in a more natural way. In this talk we review the finite element formulation of the IBM focusing, in particular, on the choice of the finite element spaces in order to guarantee a suitable mass conservation. Appropriate CFL conditions are discussed for the stability of the time marching scheme. A new implementation of the method shows a link with the *fictitious domain* method. It turns out that the new scheme enjoys more accurate mass conservation and more robust stability properties.

Key words: fluid-structure interactions, finite elements, immersed boundary method, fictitious domain, mass conservation

Mathematics subject classifications (1991): 65N30, 74F10, 65M85

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*Dipartimento di Matematica "F. Casorati", Università di Pavia, Italy, e-mail: daniele.boffi@unipv.it

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Stability analysis and finite volume element discretization for delay-driven spatial patterns in a predator-prey model.*

Raimund Bürger[†], Ricardo Ruiz-Baier[‡], Canrong Tian[§]

Abstract

Time delay is an essential ingredient of spatio-temporal predator-prey models since the reproduction of the predator population after predating the prey will not be instantaneous, but is mediated by a constant time lag accounting for the gestation of predators. Specifically, time delay is considered within a predator-prey reaction-diffusion system [12]. A stability analysis involving Hopf bifurcations [6] with respect to the delay parameter and simulations produced by a new numerical method reveal how this delay affects the formation of spatial patterns in the distribution of the species. In particular, it turns out that the delay can induce spatial patterns when the carrying capacity of the prey is large. The numerical method consists in a finite volume element (FVE) spatial discretization of the model combined with a Runge-Kutta scheme for its time discretization. FVE methods have historically been applied for flow equations [3, 4, 7, 10] and recently for several applicative time-dependent convection-diffusion problems [2, 5, 8, 9, 11]. This presentation is based on [1].

Key words: spatial patterns, time delay, pattern selection, finite volume element discretization

Mathematics subject classifications (2000): 35B35, 35B40, 65M60, 92D40.

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[†]CI^{2}MA and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: rburger@ing-mat.udec.cl

[‡]CRET-FGSE, Géopolis, Quartier UNIL-Mouline, University of Lausanne, CH-1015 Lausanne, Switzerland, e-mail: ricardo.ruizbaier@unil.ch

[§]Department of Basic Sciences, Yancheng Institute of Technology, Yancheng 224003, China, e-mail: tiancanrong@163.com

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Rate optimality of adaptive algorithms: An axiomatic approach.*

Carsten Carstensen†, Michael Feischl‡, Dirk Praetorius§

Abstract

The impact of adaptive mesh-refinement in computational partial differential equations cannot be overestimated, and convergence with optimal rates has mathematically been proved for certain model problems. We aim at a simultaneous axiomatic presentation of the proof of optimal convergence rates for adaptive finite element methods as well as boundary element methods in the spirit of [1]. For this purpose, an overall set of four axioms is sufficient and (partially even) necessary. Compared to the state of the art in the temporary literature [2, 4, 5], the improvements can be summarized as follows: First, a general framework is presented which covers the existing literature on rate optimality of adaptive schemes for both, linear as well as nonlinear problems, which is fairly independent of the underlying finite element or boundary element method. Second, efficiency of the error estimator is neither needed to prove convergence nor quasi-optimal convergence behavior of the error estimator. Instead, efficiency exclusively characterizes the approximation classes involved in terms of the bestapproximation error plus data resolution. In particular, the constraint on optimal marking parameters does not depend on the efficiency constant. Third, some general quasi-Galerkin orthogonality is not only sufficient, but also necessary for the R-linear convergence of the error estimator, which is a fundamental ingredient in the current quasi-optimality analysis [1, 2, 3, 4, 5]. Finally, the general analysis allows for equivalent error estimators and inexact solvers as well as different non-homogeneous and mixed boundary conditions.

Key words: finite element method, boundary element method, a posteriori error estimate, adaptive algorithm, convergence, optimality

Mathematics subject classifications (1991): 65N30, 65N15, 65N38

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†Humboldt-University of Berlin, Department of Mathematics, 10099 Berlin, Germany, e-mail: cc@math.hu-berlin.de

‡Vienna University of Technology, Institute for Analysis and Scientific Computing, 1040 Vienna, Austria, e-mail: Michael.Feischl@tuwien.ac.at

§Vienna University of Technology, Institute for Analysis and Scientific Computing, 1040 Vienna, Austria, e-mail: Dirk.Praetorius@tuwien.ac.at

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Analysis of a mixed finite element method for the Stokes problem with varying density in pseudostress-velocity formulation.*

Sergio Caucao[†] David Mora[‡] Ricardo Oyarzúa[§]

Abstract

We propose and analyse a mixed finite element method for the nonstandard pseudostress-velocity formulation of the Stokes problem with varying density ρ in $\mathbb{R}^d$, $d \in \{2, 3\}$. Since the resulting variational formulation does not have the standard dual-mixed structure, we reformulate the continuous problem as an equivalent fixed-point problem. Then, we apply the classical Babuška-Brezzi theory to prove that the associated mapping $\mathbb{T}$ is well defined, and assuming that $\|\frac{\nabla\rho}{\rho}\|_{\mathbf{L}^\infty(\Omega)}$ is sufficiently small, we show that $\mathbb{T}$ is a contraction mapping, which implies that the variational formulation is well-posed. Under the same hypothesis on ρ we prove stability of the continuous problem. Next, adapting to the discrete case the arguments of the continuous analysis, we are able to establish suitable hypotheses on the finite element subspaces ensuring that the associated Galerkin scheme becomes well-posed. A feasible choice of subspaces is given by Raviart-Thomas elements of order $k \geq 0$ for the pseudostress and polynomials of degree k for the velocity. Finally, several numerical results illustrating the good performance of the method with these discrete spaces, and confirming the theoretical rate of convergence, are provided.

Key words: mixed finite elements, Stokes equation

Mathematics subject classifications: 65N15, 65N30, 65N50, 74B05

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[†]Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile, email: scaucao@ubiobio.cl

[‡]Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile and **CI^{2}MA**, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: dmora@ubiobio.cl

[§]Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile and **CI^{2}MA**, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: royarzua@ubiobio.cl

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Numerical simulation of thermally convective viscoplastic fluids by semismooth second order type methods.*

Juan Carlos de los Reyes,† Sergio González-Andrade‡

Abstract

This talk is focused on the numerical solution of thermally convective viscoplastic fluids with yield stress. Following [4], a Bousinessq approximation of the convection effect is considered. The resulting coupled model is then regularized by means of a local regularization technique (see [2, 1]). We discuss a discretization in space by using a finite difference approach based on MAC scheme on staggered grids (see [3]). After space discretization, a second order BDF method is used for the time discretization of the regularized problem, leading, in each time iteration, to a nonsmooth system of equations, which is amenable to be solved by generalized Newton methods (see [2]). A semismooth Newton algorithm with a modified Jacobian is constructed for the solution of the discrete systems. Finally, we present a detailed computational experiment that exhibits the main properties of the numerical approach.

Key words: Thermal convection, viscoplastic fluids, yield stress, BDF methods, Semismooth Newton methods.

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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†ModeMat: Centro de Modelización Matemática, Escuela Politécnica Nacional, Quito, Ecuador, e-mail: juan.delosreyes@epn.edu.ec

‡ModeMat: Centro de Modelización Matemática, Escuela Politécnica Nacional, Quito, Ecuador, e-mail: sergio.gonzalez@epn.edu.ec

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On the use of second order information for the numerical solution of PDE-constrained optimization problems with sparsity. *

Juan Carlos de los Reyes[†], Estefana Loayza[‡], Pedro Merino[§]

Abstract

We present a family of algorithms for the numerical solution of PDE-constrained optimization problems, which involves an L^1-term in the objective functional. It is well known that this non-differentiable term leads to a sparse structure of the optimal control, which acts on "small" regions on the domain. In order to cope with the non-differentiability, we consider a Huber regularization of the L^1-term, which approximates the original problem by a family of parameterized differentiable problems. The general scheme of our algorithms is based on the BFGS algorithm to approximate the regular part of the cost functional. The main idea of our method is to compute descent directions by incorporating second order information. Subsequently, an orthantwise-direction strategy is used in the spirit of OW-algorithms in order to obtain a fast identification of the active sets. We present several experiments to illustrate the efficiency of our numerical algorithm.

Key words: PDE-constrained optimization, Sparsity, Second order methods.

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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[†]**ModeMat** and Departamento de Matemática, EPN-Quito, e-mail: juan.delosreyes@epn.edu.ec
[‡]**ModeMat** and Departamento de Matemática, EPN-Quito
[§]**ModeMat** and Departamento de Matemática, EPN-Quito, e-mail: pedro.merino@epn.edu.ec

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Greedy-type algorithms based on finite element discretizations approximating elliptic PDE on cartesian product domains.*

Leonardo E. Figueroa†

Abstract

We study the convergence of a variant of the separated representation method for the approximation of a class of high-dimensional PDE of self-adjoint type posed in Cartesian product domains. This method, roughly speaking, proceeds by iteratively adding to a previous iterate a tensor-product function such that the resulting new iterate has minimal residual. Here we analyze a variant of the method whose residual minimization ansatz set is the manifold of tensor products of finite element spaces—more flexible and closer to what is used in practice than the spectral method-based variants previously analyzed.

Key words: High-dimensional PDE, finite element methods, greedy algorithms

Mathematics subject classifications (1991): 65N15, 65D15, 65N30, 41A63, 41A25

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†Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: leonardo.figueroa@ci2ma.udec.cl

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New developments on the coupling of mixed-FEM and BEM for the three-dimensional Stokes problem.*

Gabriel N. Gatica† George C. Hsiao‡
Salim Meddahi§ Francisco-Javier Sayas¶

Abstract

In this paper we study the coupling of a dual-mixed variational formulation, in which the velocity, the pressure and the stress are the main unknowns, with the boundary integral equation method for the three dimensional Stokes problem. In particular, following a similar analysis given recently for the Laplacian, we are able to extend the classical Johnson & Nédélec procedure to the present case, without assuming any restrictive smoothness requirement on the coupling boundary, but only Lipschitz-continuity. More precisely, after using the incompressibility condition to eliminate the pressure, we consider the resulting velocity-stress approach with a Neumann boundary condition on an annular bounded domain, and couple the underlying equations with the single boundary integral equation arising from the application of the normal trace to the Green representation formula in the exterior unbounded region. As a result, we obtain a saddle point operator equation, which is then analyzed by the well-known Babuška-Brezzi theory. We prove the well-posedness of the continuous formulation, identifying previously the space of solutions of the associated homogeneous problem, and give explicit finite element and boundary element subspaces guaranteeing the stability of the respective Galerkin scheme. The Costabel & Han coupling procedure is also considered, and corresponding results are provided as well.

Key words: mixed-FEM, BEM, 3D Stokes problem, Johnson & Nédélec approach

Mathematics subject classifications (1991): 65N30, 65N38, 76D07, 76M10, 76M15

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†CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: ggatica@ci2ma.udec.cl

‡Department of Mathematical Sciences, University of Delaware, Newark, DE 19716-2553, USA, e-mail: hsiao@math.udel.edu

§Departamento de Matemáticas, Facultad de Ciencias, Universidad de Oviedo, Calvo Sotelo s/n, Oviedo, España, e-mail: salim@uniovi.es

¶Department of Mathematical Sciences, University of Delaware, Newark, DE 19716-2553, USA, e-mail: fjsayas@math.udel.edu.

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Dispersive and dissipative errors in the DPG method with scaled norms for Helmholtz equation.*

Jay Gopalakrishnan[†], Ignacio Muga[‡], Nicole Olivares[§]

Abstract

We consider the discontinuous Petrov-Galerkin (DPG) method, where the test space is normed by a modified graph norm. The modification scales the L^2 term of the graph norm introduced in [1], by an arbitrary positive scaling parameter $\varepsilon > 0$. The obtained method, referred to throughout as the DPG_ε method, is applied to the Helmholtz equation. We find that better results are achieved, under some circumstances, as the scaling parameter approaches the limiting value of zero. We provide an analytical understanding of this phenomenon. Next, following [2], we perform a dispersion analysis on the multiple interacting stencils that form the DPG_ε method in its lowest order setting. The analysis shows that the discrete wavenumbers of the method are complex, explaining the numerically observed artificial dissipation in the computed wave approximations. Since every DPG method is a nonstandard least-squares Galerkin method [3], its performance is compared with a standard least-squares, and other methods having a similar stencil size.

Key words: least-squares, dispersion, dissipation, quasioptimality, resonance, stencil

Mathematics subject classifications (2010): 65N30, 35J05

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[†]Fariborz Maseeh Department of Mathematics + Statistics, Portland State University, Portland, OR 97201, US, e-mail: gjay@pdx.edu

[‡]Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Valparaíso, Chile, e-mail: ignacio.muga@ucv.cl

[§]Fariborz Maseeh Department of Mathematics + Statistics, Portland State University, Portland, OR 97201, US, e-mail: nmo@pdx.edu

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Max-norm stability of low order Taylor-Hood elements in three dimensions.

Johnny Guzmán*, Manuel Sánchez-Uribe†

Abstract

We prove stability in $W^{1,\infty}(\Omega)$ and $L^{\infty}(\Omega)$ for the velocity and pressure approximations, respectively, using the lowest-order Taylor-Hood finite element spaces to solve the three dimensional Stokes problem. The domain Ω is assumed to be a convex polyhedra.

Key words: maximum norm, finite element, optimal error estimates, Stokes.

Mathematics subject classifications (1991): 65N30, 65N15.

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*Division of Applied Mathematics, Brown University, Providence, RI 02912 , e-mail: johnny_guzman@brown.edu
†Division of Applied Mathematics, Brown University, Providence, RI 02912 , e-mail: manuel_sanchez_uribe@brown.edu

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Analytical and numerical methods in shape optimization. *

Helmut Harbrecht†

Shape optimization is indispensable for designing and constructing industrial components. Many problems that arise in application, particularly in structural mechanics and in the optimal control of distributed parameter systems, can be formulated as the minimization of functionals defined over a class of admissible domains. The present talk aims at surveying on shape optimization. Especially, the following items will be addressed:

- analysis of shape optimization problems
- the discretization of shapes
- first and second order shape optimization methods
- existence and convergence of approximate shapes
- efficient numerical techniques to compute the state equation

Key words: shape optimization, shape calculus, existence and convergence of approximate solutions, optimality conditions

Mathematics subject classifications (1991): 49Q10, 49K20, 49M15, 65K10

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†Departement of Mathematics and Computer Science, University of Basel, Rheinsprung 21, 4051 Basel, Switzerland, e-mail: helmut.harbrecht@unibas.ch

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On the equivalence of fractional-order Sobolev semi-norms. *

Norbert Heuer †

Abstract

Finite and boundary element approximation error analysis consists mainly in two parts: the Bramble-Hilbert lemma and scaling properties under affine transformations of semi-norms. In the case of problems with singularities often solutions are measured by fractional-order Sobolev regularity, and there is no unique way to define semi-norms. Scaling properties are essential also in other areas of numerics, e.g., the analysis of preconditioners. Depending on the analytical setup, different definitions are useful in different situations and the important question of equivalence of semi-norms appears. In this talk we present three definitions of Sobolev semi-norms of orders between zero and one, and study their equivalence.

Key words: finite and boundary element error analysis, Sobolev spaces, semi-norms, affine transformations

Mathematics subject classifications (1991): 46E35, 47A30, 65N38

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†Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Avenidad Vicuña Mackenna 4860, Santiago, Chile, e-mail: nheuer@mat.puc.cl

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

DPG boundary elements with optimal test functions on surfaces.

Norbert Heuer, * Michael Karkulik†

Abstract

We present an ultra-weak formulation of a hypersingular integral equation on polyhedral surfaces and prove its well-posedness and equivalence with the standard variational formulation. Based on this ultra-weak formulation we present a discontinuous Petrov-Galerkin method with optimal test functions and prove its optimal convergence. The two-dimensional case on (closed) polygons has been studied in [1]. In that situation, appearing derivatives are with respect to the arc length, and Sobolev spaces are only of the L^2 and H^1-type. In this talk we study, in particular, open surfaces, where surface differential operators appear and where singularities in the exact solution prohibit to use simple L^2 and H^1 spaces for the ultra-weak formulation, though spaces of orders $\pm 1/2$ are avoided throughout. Some numerical results are shown that underline our theoretical estimates.

Key words: boundary element method, discontinuous Petrov-Galerkin method, optimal test functions

Mathematics subject classifications (1991): 65N30

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*Pontificia Universidad Católica de Chile, e-mail: nheuer@mat.puc.cl
†Pontificia Universidad Católica de Chile, e-mail: mkarkulik@mat.puc.cl

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Numerical approximation of a sorption-coagulation equation. *

Erwan Hingant,† Mauricio Sepúlveda‡

Abstract

The purpose of this talk is to present part of recent works on a sorption-coagulation equation [1]. Such equation applies to a class of water-soluble polymers that interact with metal ions. Among various possible fields of application, we find environmental science, where these polymers can be used to remove pollutant from aqueous solutions. Our objective here is to study the equation that accounts for the configurational density of polymers $f(t,p,r)$ at time t and configuration $(p,r) \in (0,+\infty)\times(0,1)$ where p stands for the size of the polymer and r the fraction of size occupied by metal ions. Briefly, the equation reads for all time $t>0$:

$$\partial_t f + \partial_r\left(\mathcal{V} f\right) = Q(f,f)\,,$$

where Q is the coagulation operator, $\mathcal{V} = k(p,r)u(t) - l(p,r)$ is the sorption rate and $u(t)$ is the concentration of free metal ions satisfying a constraint given by a balance of mass. After introducing the well-posedness of this problem, we focus on its discretization by an Euler explicit in time finite volume scheme in the spirit of [2, 3]. We will present the techniques used to prove a weak stability principle in L^1 for the sequence of approximations. Finally, a preliminary result for the a convergence will be given.

Key words: sorption coagulation equation, finite volume scheme, weak stability, convergence, existence of solutions.

Mathematics subject classifications: 65R20, 82C05, 35Q82, 35Q92

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†**CI^{2}MA**, Universidad de Concepción, Chile. E-mail: ehingant@ci2ma.udec.cl

‡**CI^{2}MA** and DIM, Universidad de Concepción, Chile.

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Local multiple traces formulation: theoretical extensions and novel applications.*

Carlos Jerez-Hanckes†

Abstract

We discuss new theoretical extensions and novel applications of the local Multiple Traces Formulation (MTF) introduced by Hiptmair & Jerez-Hanckes in 2012 [1]. The MTF was originally introduced as a set of Boundary Integral Equations to solve scattering problems for scatterers composed of heterogeneous structures with piecewise constant parameters. Numerical results were presented for 2D low-order local discretizations. In this talk, we will explore extensions of the MTF formalism to: (i) solve low and high-frequency 3D structures possessing screens, (ii) describe a domain decomposition algorithm; as well as (iii) analyze its application to neural propagation problems by coupling with Hodgin-Huxley equations and using implicit Euler and fixed point schemes.

Key words: boundary integral equations, wave scattering, Hodgin-Huxley equations

Mathematics subject classifications (2010): 65N12, 65N38, 65R20

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†Departament of Electrical Engineering, School of Engineering, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, Macul, Santiago, Chile, e-mail: cjerez@ing.puc.cl

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Finite element approximation of the eigenvalue problem for the **curl** operator in multiply connected domains.*

Eduardo Lara, † Rodolfo Rodríguez,† Pablo Venegas†

Abstract

In a recent paper [1], two of the authors introduced and analyzed a couple of numerical methods based on Nédélec finite elements to solve the eigenvalue problem for the **curl** operator in simply connected domains. This topological assumption is not just a technicality, since the eigenvalue problem is ill-posed on multiply connected domains, in the sense that its spectrum is the whole complex plane, as is shown in [2]. However, additional constraints can be added to the eigenvalue problem in order to recover a well posed problem with a discrete spectrum [2, 3]. We choose as additional constraints a zero-flux condition of the curl on all the cutting surfaces. We introduce two weak formulations of the corresponding problem, which are convenient variations of those studied in [1]; one of them is mixed and the other a Maxwell-like formulation. We prove that both are well posed and show how to modify the finite element discretization from [1] to take care of these additional constraints. We prove spectral convergence of both discretization as well as a priori error estimates. Finally, we report a numerical test which allows assessing the performance of the proposed methods.

Key words: eigenvalue problems, topological constraints, finite element methods, spectral approximation.

Mathematics subject classifications (2010): Primary 65N15, 65N25, 65N30.

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†CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mails: elara@udec.cl, rodolfo@ing-mat.udec.cl, pvenegas@ing-mat.udec.cl

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

A finite element analysis of a pseudostress formulation for the Stokes eigenvalue problem.*

Salim Meddahi[†], David Mora[‡], Rodolfo Rodríguez[§]

Abstract

In this work we analyze a finite element approximation of the Stokes eigenvalue problem. We present a variational formulation of the problem relying only on the pseudostress tensor. We present an $H(div)$-conforming discretization of the problem by means of the lowest order Brezzi-Douglas-Marini mixed finite element. We show that the resulting scheme provides a correct approximation of the spectrum and prove optimal error estimates. Finally, we present some numerical experiments supporting our theoretical results.

Key words: Stokes equations; eigenvalue problem, finite elements, error estimates.

Mathematics subject classifications (1991): 65N25, 76D07, 65N30.

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[†]Departamento de Matemáticas, Facultad de Ciencias, Universidad de Oviedo, Calvo Sotelo s/n, Oviedo, España, e-mail: salim@uniovi.es

[‡]Departamento de Matemática, Universidad del Bío-Bío and CI^{2}MA, Universidad de Concepción, Concepción, Chile, e-mail: dmora@ubiobio.cl

[§]CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, Chile, e-mail: rodolfo@ing-mat.udec.cl

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Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Numerical approximation of entropy measure valued solutions of hyperbolic conservation laws.*

Siddhartha Mishra†

Abstract

We provide substantial numerical evidence to suggest that state of the art numerical schemes may not converge to an entropy solution of systems of hyperbolic conservation laws in several space dimensions. Furthermore, entropy solutions may not be stable. Given this, we propose an extended concept of entropy measure valued solutions where the solutions are no longer functions but Young measures. We provide sufficient conditions for numerical schemes that ensure convergence to entropy measure valued solutions. Statistics of space-time averages of the measure valued solution are computed using a Monte Carlo sampling. We provide extensive numerical evidence to support our theory and to advocate that the notion of entropy measure valued solutions are an appropriate solution framework for hyperbolic conservation laws.

Key words: Conservation laws, Young measures, finite differences, finite volumes, Discontinuous Galerkin, Monte Carlo.

Mathematics subject classifications (1991): 65M06, 35L65.

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†**Seminar for Applied Mathematics (SAM), D-MATH, ETH Zurich, Switzerland e-mail: smishra@sam.math.ethz.ch**

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Well-posedness and convergence of a primal-dual hybrid discontinuous Galerkin scheme for porous media in 2-D.*

Fernando A. Morales†

Abstract

Given a simply connected polygonal domain necessary and sufficient conditions on triangulations of the region are given in order to apply a particular mixed formulation of the porous media problem [9]. The formulation allows to model the problem in function spaces of velocity and pressure which are fully decoupled across the interfaces of the triangulation. Therefore, using an edge-wise balance of the normal flux and normal stress, discontinuities of velocity and pressure across the interfaces can be introduced simultaneously while the well-posedness of the mixed formulation is ensured. The convergence of the discontinuous solution to the continuous one will be shown as well as the rate of convergence in terms of the size of the mesh. Finally, some aspects on the domain gridding will be presented, especially for the analysis of the three dimensional case.

Key words: domain decomposition, coupled discontinuous Darcy system, mixed formulations.

Mathematics subject classifications (2000): Primary 35J56, 65N55; Secondary 80A, 20, 35F15.

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†Escuela de Matemáticas, Facultad de Ciencias, Universidad Nacional de Colombia, Sede Medellín. Colombia, e-mail: famoralesj@unal.edu.co

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VALPARAISO NUMERICO IV
Séptimo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

A conforming mixed finite element method for the Navier-Stokes/Darcy coupled problem.*

Ricardo Oyarzúa†

Abstract

In this paper we introduce and analyze a mixed finite element method for the coupling of fluid flow with porous media flow. Flows are governed by the Navier-Stokes and Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We consider the standard mixed formulation in the Navier-Stokes domain and the dual-mixed one in the Darcy region, which yields the introduction of the trace of the porous medium pressure as a suitable Lagrange multiplier. We use a classical fixed point argument to prove existence and uniqueness of solution of the coupled problem under a smallness assumption on the data. The finite element subspaces defining the discrete formulation employ Bernardi-Raugel and Raviart-Thomas elements for the velocities, piecewise constants for the pressures, and continuous piecewise linear elements for the Lagrange multiplier. Similarly to the continuous case, we show stability and well-posedness of the discrete problem. In addition, the a priori error estimate for the associated Galerkin scheme is provided for small data. This talk is based on joint work with Marco Discacciati from Laboratory of Computational Methods and Numerical Analysis, Department of Applied Mathematics III, Universitat Politècnica de Catalunya, E-08034, Barcelona, Spain.

Key words: fluid flow, porous media, Navier-Stokes equation, Darcy equation

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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†GIMNAP-Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile and **CI^{2}MA**, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `royarzua@ubiobio.cl`

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Numerical solution of a multidimensional sedimentation problem using finite volume-element methods.*

Ricardo Ruiz-Baier[†], Héctor Torres[‡]

Abstract

We are interested in the reliable simulation of the sedimentation of monodisperse suspensions under the influence of body forces observed in many engineering applications and natural systems. At the macroscopic level, the complex interaction between the immiscible fluid and the sedimentation of a compressible phase may be governed by the Navier-Stokes equations coupled to a nonlinear advection-diffusion-reaction equation for the local solids concentration. We propose a versatile and effective finite volume element (FVE) scheme, whose formulation relies on a stabilized finite element (FE) method with continuous piecewise linear approximation for velocity, pressure and concentration.

Key words: Finite volume element method, Sedimentation-consolidation process, Navier-Stokes equations, Inclined channels

2000 MSC: 65M60, 65M08, 35Q35, 76D07.

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[†]CRET - FGSE, University of Lausanne, Géopolis UNIL-Mouline, CH-1015 Lausanne, Switzerland, e-mail: ricardo.ruizbaier@unil.ch

[‡]Departamento de Matemáticas, Facultad de Ciencias, Universidad de La Serena, Cisternas 1200, La Serena, Chile, e-mail: htorres@userena.cl

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On the robustness of a hybridizable discontinuous Galerkin method for curved domains.

Manuel Solano*, Bernardo Cockburn†

Abstract

A technique for solving Dirichlet-boundary value problems in curved domains was introduced in [1] for the pure diffusive case. The domain is approximated by a polygonal subdomain and the boundary condition is transferred to the computational boundary by using suitable defined extension operators. Since the computational domain is polygonal, a hybridizable discontinuous Galerkin method (HDG) was implemented to approximate the solution. Later, [2] obtained optimal error estimates for this technique under assumptions on the distance, d, between the boundary and the computational domain. In this work we present numerical evidence suggesting that, if d is of order $h/(k+1)^2$, the method is robust with respect to the meshsize h and the polynomial degree k. In addition, for convection-diffusion problems, the method is also robust if d is of order $\min\{h, Pe^{-1}\}/(k+1)^2$, where Pe is the Péclet number.

Key words: hybridizable discontinuous Galerkin, curved domains

Mathematics subject classifications: 65N30

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*$\mathbf{CI^2MA}$ and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Concepción, Chile, e-mail: msolano@ing-mat.udec.cl

†School of Mathematics, University of Minnesota, Minneapolis, USA e-mail: cockburn@math.umn.edu

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Lagrangian-remap schemes for multi-species kinematic flow problems.

Luis M. Villada[*], Raimund Bürger[†], Christophe Chalons[‡]

Abstract

The multiclass Lighthill-Whitham-Richards (MCLWR) traffic model, which distinguishes N classes of drivers differing in preferential velocity and the sedimentation of a polydisperse suspension of small rigid equal-density spheres that belong to a nite number N of species differing in size gives rise to a system of N strongly coupled, nonlinear first-order conservation laws for the local car densities or consentrations as a function of distance or depth and time. We propose a new class of anti-diffusive schemes by splitting the system of conservation laws into two different first-order quasilinear systems, the scheme is to combine the solution of the equations in a Lagrangian reference frame with an algorithm to remap the original mesh. The new schemes are addressed as Lagrangian-Remap (LR) schemes. One version of LR schemes incorporates recent anti-diffusive techniques for transport equations. The corresponding subclass of LR schemes are named Lagrangian-antidiffusive-remap(L-AR) schemes. Alternatively, the remap step can be handled by a Glimm-like random sampling method, which gives rise to a statistically conservative Lagrangian-random sampling (L-RS) scheme that is less diffusive than other remap techniques. The LR schemes for the MCLWR model are supported by a partial analysis of the L-AR schemes for $N = 1$, which are total variation diminishing (TVD) under a suitable CFL condition and therefore converge to a weak solution. Numerical examples for both L-AR and L-RS subclasses of schemes applied to MCLWR model and polydisperse sedimentation are presented.

Key words: Anti-diffusive scheme, Lagrangian-projection, system of conservation laws.

Mathematics subject classifications (1991): 65M06, 35L40, 35L45, 76T20.

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[*] **CI^{2}MA** and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: lmvillada@ing-mat.udec.cl

[†] **CI^{2}MA** and Departamento de Ingeniería Matemática, Facultad de Ciencias Físicas y Matemáticas, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: lmvillada@ing-mat.udec.cl

[‡] Laboratoire de Mathématiques de Versailles, Université de Versailles Saint-Quentinen-Yvelines, e-mail: christophe.chalons@uvsq.fr

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