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Lagrangian-Remap schemes for Multi-Species Kinematic flow problems

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Abstract

The multiclass Lighthill-Whitham-Richards (MCLWR) traffic model, which distinguishes N classes of drivers differing in preferential velocity and the sedimentation of a polydisperse suspension of small rigid equal-density spheres that belong to a nite number N of species differing in size gives rise to a system of N strongly coupled, nonlinear first-order conservation laws for the local car densities or consentrations as a function of distance or depth and time. We propose a new class of anti-diffusive schemes by splitting the system of conservation laws into two different first-order quasilinear systems, the scheme is to combine the solution of the equations in a Lagrangian reference frame with an algorithm to remap the original mesh. The new schemes are addressed as Lagrangian-Remap (LR) schemes. One version of LR schemes incorporates recent anti-diffusive techniques for transport equations. The corresponding subclass of LR schemes are named Lagrangian-antidiffusive-remap(L-AR) schemes. Alternatively, the remap step can be handled by a Glimm-like random sampling method, which gives rise to a statistically conservative Lagrangian-random sampling (L-RS) scheme that is less diffusive than other remap techniques. The LR schemes for the MCLWR model are supported by a partial analysis of the L-AR schemes for $N = 1$, which are total variation diminishing (TVD) under a suitable CFL condition and therefore converge to a weak solution. Numerical examples for both L-AR and L-RS subclasses of schemes applied to MCLWR model and polydisperse sedimentation are presented.

Key words: Anti-diffusive scheme, Lagrangian-projection, system of conservation laws.

Mathematics subject classifications (1991): 65M06, 35L40, 35L45, 76T20.

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