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A conforming mixed finite element method for the Navier-Stokes/Darcy coupled problem*

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Abstract

In this paper we introduce and analyze a mixed finite element method for the coupling of fluid flow with porous media flow. Flows are governed by the Navier-Stokes and Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We consider the standard mixed formulation in the Navier-Stokes domain and the dual-mixed one in the Darcy region, which yields the introduction of the trace of the porous medium pressure as a suitable Lagrange multiplier. We use a classical fixed point argument to prove existence and uniqueness of solution of the coupled problem under a smallness assumption on the data. The finite element subspaces defining the discrete formulation employ Bernardi-Raugel and Raviart-Thomas elements for the velocities, piecewise constants for the pressures, and continuous piecewise linear elements for the Lagrange multiplier. Similarly to the continuous case, we show stability and well-posedness of the discrete problem. In addition, the a priori error estimate for the associated Galerkin scheme is provided for small data. This talk is based on joint work with Marco Discacciati from Laboratory of Computational Methods and Numerical Analysis, Department of Applied Mathematics III, Universitat Politècnica de Catalunya, E-08034, Barcelona, Spain.

Key words: fluid flow, porous media, Navier-Stokes equation, Darcy equation

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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