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Dispersive and dissipative errors in the DPG method with scaled norms for Helmholtz equation*

Jay Gopalakrishnan[†], Ignacio Muga[‡], Nicole Olivares[§]

Abstract

We consider the discontinuous Petrov-Galerkin (DPG) method, where the test space is normed by a modified graph norm. The modification scales the L^2 term of the graph norm introduced in [1], by an arbitrary positive scaling parameter $\varepsilon > 0$. The obtained method, referred to throughout as the DPG$_\varepsilon$ method, is applied to the Helmholtz equation. We find that better results are achieved, under some circumstances, as the scaling parameter approaches the limiting value of zero. We provide an analytical understanding of this phenomenon. Next, following [2], we perform a dispersion analysis on the multiple interacting stencils that form the DPG$_\varepsilon$ method in its lowest order setting. The analysis shows that the discrete wavenumbers of the method are complex, explaining the numerically observed artificial dissipation in the computed wave approximations. Since every DPG method is a nonstandard least-squares Galerkin method [3], its performance is compared with a standard least-squares, and other methods having a similar stencil size.

Key words: least-squares, dispersion, dissipation, quasioptimality, resonance, stencil

Mathematics subject classifications (2010): 65N30, 35J05

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[†]Fariborz Maseeh Department of Mathematics + Statistics, Portland State University, Portland, OR 97201, US, e-mail: `gjay@pdx.edu`

[‡]Instituto de Matemáticas, Pontificia Universidad Católica de Valparaíso, Valparaíso, Chile, e-mail: `ignacio.muga@ucv.cl`

[§]Fariborz Maseeh Department of Mathematics + Statistics, Portland State University, Portland, OR 97201, US, e-mail: `nmo@pdx.edu`

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