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## Well-posedness and convergence of a primal-dual hybrid discontinuous Galerkin scheme for porous media in 2-D\*

FERNANDO A MORALES<sup>†</sup>

### Abstract

Given a simply connected polygonal domain necessary and sufficient conditions on triangulations of the region are given in order to apply a particular mixed formulation of the porous media problem [9]. The formulation allows to model the problem in function spaces of velocity and pressure which are fully decoupled across the interfaces of the triangulation. Therefore, using an edge-wise balance of the normal flux and normal stress, discontinuities of velocity and pressure across the interfaces can be introduced simultaneously while the well-posedness of the mixed formulation is ensured. The convergence of the discontinuous solution to the continuous one will be shown as well as the rate of convergence in terms of the size of the mesh. Finally, some aspects on the domain gridding will be presented, especially for the analysis of the three dimensional case.

**Key words:** domain decomposition, coupled discontinuous Darcy system, mixed formulations.

**Mathematics subject classifications (2000):** Primary 35J56, 65N55; Secondary 80A, 20, 35F15.

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<sup>†</sup>Escuela de Matemáticas, Facultad de Ciencias, Universidad Nacional de Colombia, Sede Medellín. Colombia, e-mail: famoralesj@unal.edu.co

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