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New developments on the coupling of mixed-FEM and BEM for the three-dimensional Stokes problem*

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Abstract

In this paper we study the coupling of a dual-mixed variational formulation, in which the velocity, the pressure and the stress are the main unknowns, with the boundary integral equation method for the three dimensional Stokes problem. In particular, following a similar analysis given recently for the Laplacian, we are able to extend the classical Johnson & Nédélec procedure to the present case, without assuming any restrictive smoothness requirement on the coupling boundary, but only Lipschitz-continuity. More precisely, after using the incompressibility condition to eliminate the pressure, we consider the resulting velocity-stress approach with a Neumann boundary condition on an annular bounded domain, and couple the underlying equations with the single boundary integral equation arising from the application of the normal trace to the Green representation formula in the exterior unbounded region. As a result, we obtain a saddle point operator equation, which is then analyzed by the well-known Babuška-Brezzi theory. We prove the well-posedness of the continuous formulation, identifying previously the space of solutions of the associated homogeneous problem, and give explicit finite element and boundary element subspaces guaranteeing the stability of the respective Galerkin scheme. The Costabel & Han coupling procedure is also considered, and corresponding results are provided as well.

Key words: mixed-FEM, BEM, 3D Stokes problem, Johnson & Nédélec approach

Mathematics subject classifications (1991): 65N30, 65N38, 76D07, 76M10, 76M15

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