VALPARAISO NUMERICO IV
Séptimo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

Analysis of a mixed finite element method for the Stokes problem with varying density in pseudostress-velocity formulation*

Sergio Caucao[†], David Mora[‡], Ricardo Oyarzúa[§]

Abstract

We propose and analyse a mixed finite element method for the nonstandard pseudostress-velocity formulation of the Stokes problem with varying density ρ in $\mathbb{R}^d$, $d \in \{2,3\}$. Since the resulting variational formulation does not have the standard dual-mixed structure, we reformulate the continuous problem as an equivalent fixed-point problem. Then, we apply the classical Babuška-Brezzi theory to prove that the associated mapping $\mathbb{T}$ is well defined, and assuming that $\|\frac{\nabla\rho}{\rho}\|_{\mathbf{L}^\infty(\Omega)}$ is sufficiently small, we show that $\mathbb{T}$ is a contraction mapping, which implies that the variational formulation is well-posed. Under the same hypothesis on ρ we prove stability of the continuous problem. Next, adapting to the discrete case the arguments of the continuous analysis, we are able to establish suitable hypotheses on the finite element subspaces ensuring that the associated Galerkin scheme becomes well-posed. A feasible choice of subspaces is given by Raviart-Thomas elements of order $k \geq 0$ for the pseudostress and polynomials of degree k for the velocity. Finally, several numerical results illustrating the good performance of the method with these discrete spaces, and confirming the theoretical rate of convergence, are provided.

Key words: mixed finite elements, Stokes equation

Mathematics subject classifications: 65N15, 65N30, 65N50, 74B05

*This research was partially supported by FONDECYT 11100180, FONDECYT 11121347 and Anillo ACT 1118

[†]Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile, email: scaucao@ubiobio.cl

[‡]Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile and **CI^{2}MA**, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: dmora@ubiobio.cl

[§]Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile and **CI^{2}MA**, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: royarzua@ubiobio.cl

References

[1] D.N. Arnold, J. Douglas and Ch.P. Gupta, *A family of higher order mixed finite element methods for plane elasticity.* Numerische Mathematatik, vol. 45, pp. 1-22, (1984).

[2] C. Bernardi, F. Laval, B. Métivet, and B. Pernaud-Thomas, *Finite element approximation of viscous flows with varying density.* SIAM Journal on Numerical Analysis. Vol. 29, 5, (1992) 1203-1243.

[3] F. Brezzi and M. Fortin, *Mixed and Hybrid Finite Element Methods.* Springer Series in Computational Mathematics, 15. Springer-Verlag, New York, 1991.

[4] G.N. Gatica, A. Márquez and M A. Sánchez, *Pseudostress-based mixed finite element methods for the Stokes problem in R^n with Dirichlet boundary conditions. I: A priori error analysis.* Communications in Computational Physics, vol. 12, 1, pp. 109-134 (2012).

[5] V. Girault and P.-A. Raviart, Finite Element Approximation of the Navier–Stokes Equations. Lecture Notes in Mathematics, 749. Springer-Verlag, Berlin-New York, 1979.