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Stability analysis and finite volume element discretization for delay-driven spatial patterns in a predator-prey model*

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Abstract

Time delay is an essential ingredient of spatio-temporal predator-prey models since the reproduction of the predator population after predating the prey will not be instantaneous, but is mediated by a constant time lag accounting for the gestation of predators. Specifically, time delay is considered within a predator-prey reaction-diffusion system [12]. A stability analysis involving Hopf bifurcations [6] with respect to the delay parameter and simulations produced by a new numerical method reveal how this delay affects the formation of spatial patterns in the distribution of the species. In particular, it turns out that the delay can induce spatial patterns when the carrying capacity of the prey is large. The numerical method consists in a finite volume element (FVE) spatial discretization of the model combined with a Runge-Kutta scheme for its time discretization. FVE methods have historically been applied for flow equations [3, 4, 7, 10] and recently for several applicative time-dependent convection-diffusion problems [2, 5, 8, 9, 11]. This presentation is based on [1].

Key words: spatial patterns, time delay, pattern selection, finite volume element discretization

Mathematics subject classifications (2000): 35B35, 35B40, 65M60, 92D40.

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