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## Convergent finite volume schemes for nonlocal and cross diffusion reaction equations. Applications to biology\*

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### Abstract

In this work, we consider reaction-diffusion systems with nonlocal and cross diffusion. We construct a finite volume scheme for this system, we establish existence and uniqueness of the discrete solution, and it is also showed that the scheme converges to the corresponding weak solution for the model studied. The convergence proof is based on the use of the discrete Sobolev embedding inequalities with general boundary conditions and a space-time  $L^1$  compactness argument that mimics the compactness lemma due to S. N. Kruzhkov. The first example of application is the description of three interacting species in a HP food chain structure. The second example of application corresponds to a mathematical model with cross-diffusion for the indirect transmission between two spatially distributed host populations having non-coincident spatial domains, transmission occurring through a contaminated environment. We give also, several numerical examples.

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