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Numerical approximation of entropy measure valued solutions of hyperbolic conservation laws^{*}

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Abstract

We provide substantial numerical evidence to suggest that state of the art numerical schemes may not converge to an entropy solution of systems of hyperbolic conservation laws in several space dimensions. Furthermore, entropy solutions may not be stable. Given this, we propose an extended concept of entropy measure valued solutions where the solutions are no longer functions but Young measures. We provide sufficient conditions for numerical schemes that ensure convergence to entropy measure valued solutions. Statistics of space-time averages of the measure valued solution are computed using a Monte Carlo sampling. We provide extensive numerical evidence to support our theory and to advocate that the notion of entropy measure valued solutions are an appropriate solution framework for hyperbolic conservation laws.

Key words: Conservation laws, Young measures, finite differences, finite volumes, Discontinuous Galerkin, Monte Carlo.

Mathematics subject classifications (1991): 65M06, 35L65.

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