VALPARAISO NUMERICO IV
Séptimo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Facultad de Ingeniería, Pontificia Universidad Católica de Valparaíso, Diciembre 11–13, 2013

An introduction to the virtual element method

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Abstract

The Virtual Element Method (VEM) is a very recent technology [1] for the discretization of partial differential equations. The VEM can be interpreted as a novel approach that shares the same variational background as the Finite Element Method but enjoys also a connection with modern Mimetic schemes. By avoiding the explicit integration of the shape functions that span the discrete Galerkin space and introducing a novel construction of the associated stiffness matrixes, the VEM acquires very interesting properties and advantages with respect to more standard Galerkin methods, yet still keeping the same coding complexity. For instance, the VEM easily allows for polygonal/polyhedral meshes (even non-conforming) with non-convex elements and possibly with curved faces; it allows for discrete solutions of arbitrary C^k regularity, defined on unstructured meshes. The present talk is an introduction to the VEM, aiming at showing the main ideas of the method from [1]. After introducing the method on a simple model problem, we will present an (optimal) convergence result and some numerical tests. We will moreover address the practical construction of the scheme [1, 6] and also some interesting possibilities such as that of using high regularity discrete spaces [2]. Other recent advances on the VEM, here not described, are [3, 4, 5].

Key words: Galerkin approximation, polygonal and polyhedral meshes, Virtual Element Method

Mathematics subject classifications (2010): 65N30, 65N99.

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