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Analysis of an augmented fully-mixed finite element method for a three-dimensional fluid-solid interaction problem*

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Abstract

We introduce and analyze an augmented fully-mixed finite element method for a fluid-solid interaction problem in 3D. The media are governed by the acoustic and elastodynamic equations in time-harmonic regime, and the transmission conditions are given by the equilibrium of forces and the equality of the corresponding normal displacements. We first employ dual-mixed variational formulations in both domains, which yields the Cauchy stress tensor and the rotation of the solid, together with the gradient of the pressure of the fluid, as the preliminary unknowns. This approach allows us to extend an idea from a recent own work in such a way that both transmission conditions are incorporated now into the definitions of the continuous spaces, and therefore no unknowns on the coupling boundary appear. As a consequence, the pressure of the fluid and the displacement of the solid become explicit unknowns of the coupled problem, and hence two redundant variational terms arising from the constitutive equations, both of them multiplied by stabilization parameters, need to be added for well-posedness. In fact, we show that explicit choices of the above mentioned parameters and a suitable decomposition of the spaces allow the application of the Babuška-Brezzi theory and the Fredholm alternative for concluding the solvability of the resulting augmented formulation. The unknowns of the fluid and the solid are then approximated by a conforming Galerkin scheme defined in terms of Arnold-Falk-Winther and Lagrange finite element subspaces of order 1. The analysis of the discrete method relies on a stable decomposition of the finite element spaces and also on a classical result on projection methods for Fredholm operators of index zero.

Key words: mixed finite elements, Helmholtz equation, elastodynamic equation

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 74F10, 74B05, 35J05

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