Centro de Investigación en Ingeniería Matemática

Universidad de Concepción

Second CI^{2}MA Focus Seminar
"Computational Electromagnetism"

January 5, 2012
Auditorio Alamiro Robledo
Facultad de Ciencias Físicas y Matemáticas
Universidad de Concepción

Organizer: Rodolfo Rodríguez

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Programme

15.00 **Opening**

15.05 **Ana Alonso** (Università degli Studi di Trento, Italia):
Iterative methods for the saddle-point problem arising from the $\mathbf{H}_C/\mathbf{E}_I$ formulation of the eddy current model.

15.45 **Bibiana López-Rodríguez** (CI^{2}MA, Universidad de Concepción):
Equivalence between two finite element methods for the eddy current problem.

16.15 **Jessika Camaño** (CI^{2}MA, Universidad de Concepción):
Inverse source problems for eddy current equations.

16.45 **Carlos Jerez-Hanckes** (Pontificia Universidad Católica de Chile):
Multiple traces boundary integral formulation for Helmholtz transmission problems.

17.15 **Coffee break**

17.45 **Rodolfo Rodríguez** (CI^{2}MA, Universidad de Concepción):
Mathematical and numerical analysis of an eddy current time-dependent problem on moving domains arising from electromagnetic forming

18.15 **Pablo Venegas** (CI^{2}MA, Universidad de Concepción):
Numerical solution of transient nonlinear axisymmetric eddy current models with hysteresis.

18.45 **Ralf Hiptmair** (SAM – ETH Zurich, Suiza):
Boundary element methods for eddy current computation

19.30 **Closing**

21.00 **Seminar Dinner**

Practical information

Seminar participants who would like to join dinner should register with CI^{2}MA secretary:

Ms Angelina Fritz
CI^{2}MA, office 24
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Abstracts

Iterative methods for the saddle-point problem arising from the $\mathbf{H}_C/\mathbf{E}_I$ formulation of the eddy current model

Ana Alonso[1]

The solution of the linear system arising from a finite element approximation of the time-harmonic eddy current problem is considered. In particular we focus on the formulation that retains as main unknowns the magnetic field in the conductor $\mathbf{H}_C$ and the electric field in the insulator $\mathbf{E}_I$. We propose and analyze iterative procedures based on the physical decomposition of the computational domain in a conducting region and an air region. If the conductor is simply connected we prove that the Dirichlet/Neumann iteration converges with a rate that is independent of the mesh size. In the case of general topology we propose to use either a modified version of the Dirichlet/Neumann iteration or an Uzawa-like method. We compare the performance of both methods by solving four different test problems.

This presentation is based on joint work with Rafael Vázquez-Hernández (Istituto di Matematica Applicata e Tecnologie Informatiche, Pavia, Italy).

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Equivalence between two finite element methods for the eddy current problem

Bibiana López-Rodríguez[1]

The goal of this talk is to prove that two, in principle different, well-known finite element approximations of the eddy current model are equivalent. The first one concerns a formulation involving the magnetic field in the conductor and the magnetic scalar potential in the dielectric ([2]). The second one solves another formulation of the same problem involving the magnetic field in both, the conductor and the dielectric, and a Lagrange multiplier in the dielectric ([1]). The latter is also shown to be equivalent to a third formulation involving two Lagrange multipliers (also introduced in [1], based on results from [3]), which leads to a well posed linear system.

This is a joint work with Alfredo Bermúdez and Pilar Salgado (both from Universidade de Santiago de Compostela, Spain) and Rodolfo Rodríguez (Universidad de Concepción, Chile).

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Inverse source problems for eddy current equations

Jessika Camaño[1]

We study the inverse source problem for the eddy current approximation of Maxwell equations. As for the full system of Maxwell equations, ([1, 2, 3]) we show that a volume current source cannot be uniquely identified by the knowledge of the tangential components of the electromagnetic fields on the boundary, and we characterize the space of non-radiating sources. On the other hand, we prove that the inverse source problem has a unique solution if the source is supported on the boundary of a subdomain or if it is the sum of a finite number of dipoles. We address the applicability of this result for the localization of brain activity from electroencephalography and magnetoencephalography measurements.

This is a joint work with Ana Alonso-Rodríguez and Alberto Valli (both from Università di Trento, Italy).

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Multiple traces boundary integral formulation for Helmholtz transmission problems

Carlos Jerez-Hanckes[1]

We present a novel boundary integral formulation of the Helmholtz transmission problem for bounded composite scatterers (that is, piecewise constant material parameters in "subdomains") that directly lends itself to operator preconditioning via Calderón projectors. The method relies on local traces on subdomains and weak enforcement of transmission conditions. The variational formulation is set in Cartesian products of standard Dirichlet and special Neumann trace spaces for which restriction and extension by zero are well defined. In particular, the Neumann trace spaces over each subdomain boundary are built as piecewise $\tilde{H}^{-1/2}$-distributions over each associated interface. Through the use of interior Calderón projectors, the problem is cast in variational Galerkin form with an operator matrix whose diagonal is composed of block boundary integral operators associated with the subdomains. We show existence and uniqueness of solutions based on an extension of Lions' projection lemma for non-closed subspaces. We also investigate asymptotic quasi-optimality of conforming boundary element Galerkin discretization. Numerical experiments in 2-D confirm the efficacy of the method and a performance matching that of another widely used boundary element discretization. They also demonstrate its amenability to different types of preconditioning.

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Mathematical and numerical analysis of an eddy current time-dependent problem on moving domains arising from electromagnetic forming

Rodolfo Rodríguez[1]

Electromagnetic forming is a metal working process that relies on the use of electromagnetic forces to deform metallic workpieces at high speeds. A transient electric current is induced in a coil using a capacitor bank and high-speed switches. This current induces a magnetic field that penetrates the nearby conductive workpiece where an eddy current is generated. The magnetic field, together with the eddy current, induce Lorentz forces that drive the deformation of the workpiece [1, 2].
In this work, we focus on the underlying electromagnetic model for a workpiece under the action of a coil in an axisymmetric setting [3, 4]. With this aim, we introduce a time-dependent variational formulation which leads to a degenerate parabolic problem. We establish its well-posedness by means of a regularization process. To solve the weak problem, we develop a computational code that uses finite elements for the space variable and implicit time-discretization. *A priori* error estimates for the fully-discretized problem are proved. Finally, some numerical experiments which allow assessing the performance of the method are reported.

This presentation is based on joint work with Alfredo Bermúdez, Rafael Muñoz-Sola and Pilar Salgado (all from Universidade de Santiago de Compostela, Spain) and Carlos Reales (from Universidad de Montería, Colombia).

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Numerical solution of transient nonlinear axisymmetric eddy current models with hysteresis

Pablo Venegas[1]

This work deals with the mathematical analysis and the computation of transient electromagnetic fields in nonlinear magnetic media with hysteresis. The result of this work complements the content of [1], where existence of the solution has been proved under fairly general assumptions on the **H**–**B** curve, namely, the nonlinear constitutive relation between the magnetic field **H** and the magnetic induction **B**. In our case, the constitutive relation between **H** and **B** is given by a hysteresis operator, i.e., the values of magnetic field depend not only on the present values of magnetic induction but also on the past history. Like in [1], we assume axisymmetry of the fields and consider two kinds of boundary conditions. Firstly the magnetic field is given on the boundary (Dirichlet boundary condition). Secondly, the magnetic flux through a meridional plane is given, leading to a non-standard boundary-value problem. For both problems, under suitable assumptions, an existence result is achieved. The technique we use is based on implicit time discretization, a priori estimates and passage to the limit by compactness (see, for instance, [2] and [3]). Finally we consider an application: the numerical computation of eddy current losses in laminated media as those used in transformers or electrical machines.

This is a joint work with Alfredo Bermúdez and M. Dolores Gómez (both from Universidade de Santiago de Compostela, Spain) and Rodolfo Rodríguez (Universidad de Concepción, Chile).

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Boundary element methods for eddy current computation

Ralf Hiptmair[1]

I review numerical methods for time-harmonic eddy current problems in the case of homogeneous, isotropic, and linear materials. In particular, I am concerned with approaches that entirely rely on boundary integral equations and their conforming Galerkin discretization. Starting point are both E- and H-based strong formulation, for which issues of gauging and topological constraints on the existence of potentials are discussed.
Direct boundary integral equations and the so-called symmetric coupling of the integral equations corresponding to the conductor and the non-conducting regions are employed. They give rise to coupled variational problems that are elliptic in suitable trace spaces. This implies quasi-optimal convergence of conforming Galerkin boundary element methods, which make use of divergence-conforming trial spaces for surface currents.

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