Sixth CI^{2}MA Focus Seminar

Stochastic Modeling and Numerical Analysis

August 8, 2013
Auditorio Alamiro Robledo
Facultad de Ciencias Físicas y Matemáticas
Universidad de Concepción

Organizers: Soledad Torres and Rodolfo Rodríguez

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Programme

09.00 **Opening**

09.05 **Ciprian Tudor** (Université de Lille 1, France):
Variations of the solution to the heat equation with fractional colored noise

09.45 **Soledad Torres** (CIMFAV, Universidad de Valparaíso, Chile):
Estimation of the paremeter of the skew Brownian motion

10.25 **Ricardo Castro** (Universidad del Bío Bío, Chile):
Quantum probability and measurement theory

11.05 **Coffee break**

11.35 **Carlos Mora** (CI^{2}MA, Universidad de Concepción):
Ehrenfest-type theorems for open quantum systems

12.15 **Frederi Viens** (Purdue University, USA):
Hitting probabilities for general Gaussian processes

12.55 **Closing**

13:30 **Seminar lunch**

Acknowledgement

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Practical information

Seminar participants who would like to join lunch should register with CI^{2}MA secretary:

Ms Angelina Fritz
CI^{2}MA, office 24
E-mail: afritz@ci2ma.udec.cl
Phone: (041) 266 1324

Abstracts

Variations of the solution to the heat equation with fractional colored noise

Ciprian Tudor[1]

We will discuss recent results on the existence of the solution to the heat equation driven by a Gaussian noise which behaves as a fractional Brownian motion in time and has correlated spatial structure. We will present various properties of this solution: sharp regularity of the sample paths, its law and its scaling properties. Using Malliavin calculus, we analyze the asymptotic behavior of the quadratic variations of the solution.

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Estimation of the paremeter of the skew Brownian motion

Soledad Torres[1]

We study the asymptotic behavior of the maximum likelihood estimator corresponding to the observation of a trajectory of a skew Brownian motion through a uniform time discretization. We characterize the speed of convergence and the limiting distribution when the step size goes to zero, which in this case are non-classical, under the null hypothesis of the skew Brownian motion being a usual Brownian motion. This allows us to design a test on the skewness parameter. We show that numerical simulations can be easily performed to estimate the skewness parameter and provide an application in biology.

This is a joint work with Antoine Lejay (INRIA project-team TOSCA, IECL, Université de Lorraine, Nancy, France) and Ernesto Mordecki (Centro de Matemática, Facultad de Ciencias, Universidad de la República, Montevideo, Uruguay).

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Quantum probability and measurement theory

Ricardo Castro Santis[1]

A review on the measurement theory in quantum open systems will be presented. The main mathematical tool will be the Hudson-Parathasarathy equations in the quantum open systems context. The talk will be focused mainly on the case when the coefficients of the Hudson-Parathasarathy equation are unbounded operators. An aproximation of the reduced dynamics will be showed using Markovian semigroups and the time-dependent infinitesimal generator for the dynamics. Finally, we show an application to the degenerate harmonic oscillator.

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Ehrenfest-type theorems for open quantum systems

Carlos Mora[1]

This talk is based on a joint work with Franco Fagnola [1], where we study open quantum systems with state space $L^2\left(\mathbb{R}^d,\mathbb{C}\right)$. First, we develop basic properties of quantum systems in Gorini-Kossakowski-Sudarshan-Linblad form with Hamiltonian

$$H(t)=-\alpha\Delta+i\sum_{j=1}^{d}\left(A^j(t,\cdot)\partial_j+\partial_j A^j(t,\cdot)\right)+V(t,\cdot)$$

and interaction operators

$$L_\ell\left(t\right)=\begin{cases}\sum_{j=1}^{d}\sigma_{\ell j}\left(t,\cdot\right)\partial_j+\eta_\ell\left(t,\cdot\right), & \text{if } \ 1\leq\ell\leq m,\\ 0, & \text{if } \ \ell>m.\end{cases}$$

Here $t\geq 0$, $m\in\mathbb{N}$, α is a non-negative real constant, ∂_j denotes the partial derivative with respect to the j^{th}-coordinate, $V, A^j:[0,+\infty[\times\mathbb{R}^d\to\mathbb{R}$ and $\sigma_{\ell j}, \eta_\ell:[0,+\infty[\times\mathbb{R}^d\to\mathbb{C}$ are measurable smooth functions. Then, we establish that the mean-value of the observable A at time t satisfies, roughly speaking, the Ehrenfest-type equation

$$\frac{d}{dt}\mathrm{tr}\left(\rho_t A\right)=\mathrm{tr}\left(\rho_t\left(-i\left[A,H\left(t\right)\right]+\frac{1}{2}L_\ell\left(t\right)^*\left[A,L_\ell\left(t\right)\right]+\frac{1}{2}\left[L_\ell\left(t\right)^*,A\right]L_\ell\left(t\right)\right)\right), \quad (1)$$

where A is relatively bounded with respect to $-\Delta+|x|^2$, ρ_t denotes the reduced density operator at time t, and $[\cdot,\cdot]$, resp. $\mathrm{tr}\left(\cdot\right)$, stands for the commutator between two operators, resp. the trace operation. Finally, using (1), together with its stochastic version, we study the dynamics of physical systems such as fluctuating ion traps and quantum measurement processes of position.

This is a joint work with Franco Fagnola (Dipartimento di Matematica, Politecnico di Milano, Italy).

References

[1] Fagnola, F. and Mora, C.M., Stochastic Schrödinger equations and applications to Ehrenfest-type theorems, *ALEA, Lat. Am. J. Probab. Math. Stat.*, Vol. 10, No. 1, pp. 191–223. (2013).

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Hitting probabilities for general Gaussian processes

Frederi Viens[1]

It is known that d-dimensional fractional Brownian motion (fBm) with parameter H hits points with positive probability if and only if $d < 1/H$. Very few results are available for Gaussian proceses beyond fBm. We present tools from the Malliavin calculus and a general strategy from potential theory, which help us finding upper and lower bounds on hitting probabilities for points and other sets for general Gaussian processes. We also explain what problems remain open and how non-Gaussian extensions could look like. Results are devised to be intrinsic to the processes' laws, and to avoid any explicit reference to Hölder exponents.

This is a joint work with Eulalia Nualar (Department of Economics and Business, Universitat Pompeu Fabra, Barcelona, Spain).

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