14th CI^{2}MA FOCUS SEMINAR: Finite Element and Related Methods for PDEs in Continuum Mechanics

CENTRO CI^{2}MA, UNIVERSIDAD DE CONCEPCIÓN, AUGUST 27 - 28, 2018

Supported by CI^{2}MA *and by* CONICYT-Chile *through the project* AFB170001 *of the* PIA *Program*

Organizers: Gabriel N. Gatica (UdeC) and Ricardo Oyarzúa (UBB)

PROGRAMME

LUNES, 27 DE AGOSTO (MAÑANA)

10.00 **Ernesto Castillo** (Departamento de Ingeniería Mecánica, USACH):
Dynamic term-by-term stabilized finite element formulation for the incompressible Navier-Stokes problem.

10.30 **Sergio Caucao** (CI^{2}MA, UdeC):
A conforming mixed finite element method for the Navier–Stokes/Darcy–Forchheimer coupled problem.

11.00 **Paul Méndez** (CI^{2}MA & Departamento de Ingeniería Matemática, UdeC):
On $\boldsymbol{H}(\mathrm{div})$-conforming methods for double-diffusion equations in porous media.

11.30 **David Mora** (Departamento de Matemática, Facultad de Ciencias, UBB & CI^{2}MA, UdeC):
Virtual elements for a shear-deflection formulation of Reissner-Mindlin plates.

15.00 **Ramón Codina** (Department of Civil and Environmental Engineering, UPC, Spain):
The linked Lagrange multiplier technique to prescribe essential boundary conditions of elliptic problems on embedded meshes.

15.30 **Bryan Gómez** (CI^{2}MA & Departamento de Ingeniería Matemática, UdeC):
Formulation and analysis of fully-mixed methods for stress-assisted diffusion problems.

16.00 **Iván Velásquez** (CI^{2}MA & Departamento de Ingeniería Matemática, UdeC):
Virtual element analysis for the Buckling problem of thin plates.

16.30 **Jessika Camaño** (Departamento de Matemática y Física Aplicadas, UCSC & CI^{2}MA, UdeC):
Analysis of a conservative mixed-FEM for the stationary Navier–Stokes problem.

17.00 Coffee Break

17.30 **Pablo Venegas** (Departamento de Matemática, Facultad de Ciencias, UBB):
Numerical analysis of a time-domain elastoacoustic problem.

18.00 **Felipe Lepe** (Departamento de Matemática, Facultad de Ciencias, UBB):
DG methods for spectral problems.

18.30 **Cinthya Rivas** (CI^{2}MA & Departamento de Ingeniería Matemática, UdeC):
An asymptotic model based on matching far and near fields for thin gratings problems.

19.00 **Mauricio Munar** (CI^{2}MA & Departamento de Ingeniería Matemática, UdeC):
A mixed virtual element method for the Navier-Stokes equations.

20.30 Cena de Camaradería (Restaurante Faro Belén)

Martes, 28 de Agosto (mañana)

10.00 **Mauricio Sepúlveda** (CI^2MA & Departamento de Ingeniería Matemática, UdeC):
On exponential stability for thermoelastic plates: comparison and singular limits

10.30 **Patrick Vega** (CI^2MA & Departamento de Ingeniería Matemática, UdeC):
A posteriori error analysis of HDG methods in fluid mechanics.

11.00 **Carlos García** (Departamento de Matemática, Facultad de Ciencias, UBB & CI^2MA, UdeC):
Analysis of a mixed-FEM for stationary incompressible magneto-hydrodynamics.

11.30 **Gabriel N. Gatica** (CI^2MA & Departamento de Ingeniería Matemática, UdeC):
On the coupling of VEM and BEM in two and three dimensions.

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DYNAMIC TERM-BY-TERM STABILIZED FINITE ELEMENT FORMULATION FOR THE INCOMPRESSIBLE NAVIER-STOKES PROBLEM

ERNESTO CASTILLO

Abstract. In this work, we propose and analyze the stability and the dissipative structure of a new dynamic term-by-term stabilized finite element formulation for the Navier-Stokes problem, which adds the minimum number of stabilizing terms and can be viewed as a variational multiscale (VMS) method under some assumptions. The essential point of the formulation is the time dependent nature of the subscales and, contrary to residual-based formulations, the introduction of two velocity subscale components. They represent the components of the convective and the pressure gradient terms, respectively, of the momentum equation that cannot be captured by the finite element mesh. A key idea of the proposed method is that the convective subscale is close to a solenoidal field and the pressure gradient subscale is close to a potential field. The method ensures stability in anisotropic space-time discretizations, which is proved using numerical analysis under suitable assumptions and demonstrated in classical numerical tests. The work includes a detailed description of the proposed formulation and several numerical examples that serve to justify our claims.

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A CONFORMING MIXED FINITE ELEMENT METHOD FOR THE NAVIER–STOKES/DARCY–FORCHHEIMER COUPLED PROBLEM

SERGIO CAUCAO, MARCO DISCACCIATI, GABRIEL N. GATICA, AND RICARDO OYARZÚA

Abstract. In this work we present and analyse a mixed finite element method for the coupling of fluid flow with porous media flow. The flows are governed by the Navier–Stokes and the Darcy–Forchheimer equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers–Joseph–Saffman law. We consider the standard mixed formulation in the Navier–Stokes domain and the dual-mixed one in the Darcy–Forchheimer region, which yields the introduction of the trace of the porous medium pressure as a suitable Lagrange multiplier. The well-posedness of the problem is achieved by combining a fixed-point strategy, classical results on nonlinear monotone operators and the well-known Schauder and Banach fixed-point theorems. As for the associated Galerkin scheme we employ Bernardi–Raugel and Raviart–Thomas elements for the velocities, and piecewise constant elements for the pressures and the Lagrange multiplier, whereas its existence and uniqueness of solution is established similarly to its continuous counterpart, using in this case the Brouwer and Banach fixed-point theorems, respectively. We show stability, convergence, and a priori error estimates for the associated Galerkin scheme. Finally, we report some numerical examples confirming the predicted rates of convergence, and illustrating the performance of the method.

References

[1] S. Caucao, G.N. Gatica, R. Oyarzúa and I. Šebestová, *A fully-mixed finite element method for the Navier–Stokes/Darcy coupled problem with nonlinear viscosity.* J. Numer. Math. 25 (2017), no. 2, 55–88.
[2] M. Discacciati and R. Oyarzúa, *A conforming mixed finite element method for the Navier–Stokes/Darcy coupled problem.* Numer. Math. 135 (2017), no. 2, 571–606.
[3] A. Ern and J.-L. Guermond, *Theory and Practice of Finite Elements.* Applied Mathematical Sciences, 159. Springer-Verlag, New York, 2004.
[4] V.J. Ervin, E.W. Jenkins and S. Sun, *Coupled generalized nonlinear Stokes flow with flow through a porous medium.* SIAM J. Numer. Anal. 47 (2009), no. 2, 929–952.
[5] G.N. Gatica, *A Simple Introduction to the Mixed Finite Element Method. Theory and Applications.* Springer Briefs in Mathematics. Springer, Cham, 2014.

This research was partially supported by CONICYT-Chile through BASAL project CMM, Universidad de Chile, project Fondecyt 1161325, and Becas-Chile Programme for Chilean students; by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción; and by Universidad del Bío-Bío through DIUBB project GI 171508VC.

[6] G.N. Gatica, S. Meddahi and R. Oyarzúa, *A conforming mixed finite-element method for the coupling of fluid flow with porous media flow*. IMA J. Numer. Anal. 29 (2009), no. 1, 86–108.

[7] P. Grisvard, *Théorèmes de traces relatifs à un polyèdre*. C. R. Acad. Sci. Paris Sr. A 278 (1974), 1581–1583.

[8] P. Grisvard, *Elliptic Problems in Nonsmooth Domains*. Monographs and Studies in Mathematics, 24. Pitman (Advanced Publishing Program), Boston, MA, 1985.

[9] H. Manouzi and M. Farhloul, *Mixed finite element analysis of a non-linear three-fields Stokes model*. IMA J. Numer. Anal. 21 (2001), no. 1, 143–164.

[10] H. Pan and H. Rui, *Mixed element method for two-dimensional Darcy–Forchheimer model*. J. Sci. Comput. 52 (2012), no. 3, 563–587.

[11] B. Scheurer, *Existence et approximation de points selles pour certains problèmes non linéaires*. RAIRO Anal. Numér. 11 (1977), no. 4, 369–400.

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ON $\boldsymbol{H}(\mathrm{div})$-CONFORMING METHODS FOR DOUBLE-DIFFUSION EQUATIONS IN POROUS MEDIA

RAIMUND BÜRGER, PAUL E. MÉNDEZ, AND RICARDO RUIZ BAIER

Abstract. A stationary Navier-Stokes-Brinkman model coupled to a system of advection-diffusion equations serves as a model for so-called double-diffusive viscous flow in porous media in which both heat and a solute within the fluid phase are subject to transport and diffusion. The solvability analysis of these governing equations results as a combination of compactness arguments and fixed-point theory. In addition an $\boldsymbol{H}(\mathrm{div})$-conforming discretisation is formulated by a modification of existing methods for Brinkman flows. The well-posedness of the discrete Galerkin formulation is also discussed, and convergence properties are derived rigorously. Computational tests confirm the predicted rates of error decay and illustrate the applicability of the methods for the simulation of bacterial bioconvection and thermohaline circulation problems.

References

[1] V. Anaya, M. Bendahmane, D. Mora and R. Ruiz-Baier, *On a primal-mixed vorticity-based formulation for reaction-diffusion-Brinkman systems.* Netw. Heterog. Media 13 (2018), no. 1, 69–94.
[2] D.N. Arnold, F. Brezzi, B. Cockburn and L.D. Marini, *Unified analysis of discontinuous Galerkin methods for elliptic problems.* SIAM J. Numer. Anal. 39 (2001/02), no. 5, 1749–1779.
[3] C.S. Balla and K. Naikoti, *Soret and Dufour effects on free convective heat and solute transfer in fluid saturated inclined porous cavity.* Int. J. Sci. Eng. 18 (2015), no. 4, 543–554.
[4] F. Brezzi, J. Douglas and L.D. Marini, Two families of mixed finite elements for second order elliptic problems. Numer. Math. 47 (1985), 217–235.
[5] R. Bürger, S.K. Kenettinkara, R. Ruiz-Baier and H. Torres, *Coupling of discontinuous Galerkin schemes for viscous flow in porous media with adsorption.* SIAM J. Sci. Comput. 40 (2018), no. 2, B637–B662.
[6] J. Könnö and R. Stenberg, *H(div)-conforming finite elements for the Brinkman problem.* Math. Models Methods Appl. Sci. 21 (2011), no. 11, 2227–2248.
[7] P. Lenarda, M. Paggi and R. Ruiz-Baier, *Partitioned coupling of advection-diffusion-reaction systems and Brinkman flows.* J. Comput. Phys. 344 (2017), 281–302.

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[8] C. Lin and L.E. Payne, *Continuous dependence on the Soret coefficient for double diffusive convection in Darcy flow.* J. Math. Anal. Appl. 342 (2008), 311–325.

[9] S.A. Lorca and J.L Boldrini, *Stationary solutions for generalized Boussinesq models.* J. Differ. Equ. 124 (1996), no. 2, 389–406

[10] R. Oyarzúa, T. Qin and D. Schötzau, *An exactly divergence-free finite element method for a generalized Boussinesq problem.* IMA J. Numer. Anal. 34 (2014), no. 3, 1104–1135.

[11] Q. Shao, M. Fahs, A. Younes, A. Makradi and T. Mara, *A new benchmark reference solution for double-diffusive convection in a heterogeneous porous medium.* Numer. Heat Transfer B 70 (2016), no. 5, 373–392.

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VIRTUAL ELEMENTS FOR A SHEAR-DEFLECTION FORMULATION OF REISSNER-MINDLIN PLATES

L. BEIRÃO DA VEIGA, D. MORA, AND G. RIVERA

ABSTRACT. We present a virtual element method for the Reissner–Mindlin plate bending problem which uses shear strain and deflection as discrete variables without the need of any reduction operator. The proposed method is conforming in $[H^1(\Omega)]^2 \times H^2(\Omega)$ and has the advantages of using general polygonal meshes and yielding a direct approximation of the shear strains. The rotations are then obtained by a simple postprocess from the shear strain and deflection. We prove convergence estimates with involved constants that are uniform in the thickness t of the plate. Finally, we report numerical experiments which allow us to assess the performance of the method.

REFERENCES

[1] P.F. ANTONIETTI, L. BEIRÃO DA VEIGA, S. SCACCHI AND M. VERANI, *A C^1 virtual element method for the Cahn–Hilliard equation with polygonal meshes*, SIAM J. Numer. Anal., 54 (2016), no. 1, 36–56.

[2] L. BEIRÃO DA VEIGA, F. BREZZI, A. CANGIANI, G. MANZINI, L.D. MARINI AND A. RUSSO, *Basic principles of virtual element methods*, Math. Models Methods Appl. Sci., 23 (2013), 199–214.

[3] F. BREZZI AND L.D. MARINI, *Virtual elements for plate bending problems*, Comput. Methods Appl. Mech. Engrg., 253 (2012), 455–462.

[4] D. MORA, G. RIVERA AND I. VELÁSQUEZ, *A virtual element method for the vibration problem of Kirchhoff plates*, ESAIM Math. Model. Numer. Anal., DOI: 10.1051/m2an/2017041 (2018).

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THE LINKED LAGRANGE MULTIPLIER TECHNIQUE TO PRESCRIBE ESSENTIAL BOUNDARY CONDITIONS OF ELLIPTIC PROBLEMS ON EMBEDDED MESHES

RAMON CODINA

Abstract. In this work we propose a method to prescribe essential boundary conditions in the finite element approximation of elliptic problems when the boundary of the computational domain does not match with the element boundaries. The problems considered are the Poisson problem, the Stokes problem, the Darcy problem, both in the primal and in the dual formulation, and Maxwells problem. The formulation proposed is of variational type. The key idea is to start with the variational form that defines the problem and treat the boundary condition as a constraint. The particular feature is that the Lagrange multiplier is not defined on the boundary where the essential condition needs to be prescribed, but is taken as a certain trace of a field defined in the computational domain, either in all of it or just in a region surrounding the boundary. When approximated numerically, this allows one to condense the degrees of freedom of the new field and end up with a problem posed only in terms of the original unknowns. The nature of the field used to weakly impose boundary conditions depends on the problem being treated. For the Poisson problem it is a flux, for the Stokes problem a stress, and for the Darcy problem a velocity field. In all cases, this new variable can be eliminated imposing that it is equal (in a least squares sense) to the variable whose trace is the Lagrange multiplier. The resulting problem resembles Nitsche's way to impose boundary conditions, with the advantage that no conditions on the parameter are required for stability and, in particular, that this parameter can be taken smaller in the formulation we propose.

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FORMULATION AND ANALYSIS OF FULLY-MIXED METHODS FOR STRESS-ASSISTED DIFFUSION PROBLEMS

GABRIEL N. GATICA, BRYAN GÓMEZ-VARGAS, AND RICARDO RUIZ-BAIER

ABSTRACT. This paper is devoted to the mathematical and numerical analysis of a mixed-mixed PDE system describing the stress-assisted diffusion of a solute into an elastic material. The equations of elastostatics are written in mixed form using stress, rotation and displacements, whereas the diffusion equation is also set in a mixed three-field form, solving for the solute concentration, for its gradient, and for the diffusive flux. This setting simplifies the treatment of the nonlinearity in the stress-assisted diffusion term. The analysis of existence and uniqueness of weak solutions to the coupled problem follows as combination of Schauder and Banach fixed-point theorems together with the Babuška-Brezzi and Lax-Milgram theories. Concerning numerical discretization, we propose two families of finite element methods, based on either PEERS or Arnold-Falk-Winther elements for elasticity, and a Raviart-Thomas and piecewise polynomial triplet approximating the mixed dif- fusion equation. We prove the well-posedness of the discrete problems, and derive optimal error bounds using a Strang inequality. Finally, numerical experiments illustrate the interest of this scheme and validate the theory.

REFERENCES

[1] E.C. AIFANTIS, *On the problem of diffusion in solids.* Acta Mech. 37 (1980), 265–296.
[2] C. BACUTA AND J.H. BRAMBLE, *Regularity estimates for solutions of the equations of linear elasticity in convex plane polygonal domains.* Z. Angew. Math. Phys. 54 (2003), no. 5, 874–878.
[3] G.N. GATICA, B. GOMEZ-VARGAS AND R. RUIZ-BAIER, *Analysis and mixed-primal finite element discretisations for stress-assisted diffusion problems.* Comput. Methods Appl. Mech. Engrg. (2018), no. 337, 411–438.
[4] G.N. GATICA, B. GOMEZ-VARGAS AND R. RUIZ-BAIER, *Formulation and analysis of fully-mixed methods for coupled stress-diffusion problems.* Preprint 2018-08, Centro de Investigacion en Ingenieria Matematica (CI^{2}MA), Universidad de Concepcion, Chile, (2018).
[5] P. GRISVARD, *Smoothness of the solution of a monotonic boundary value problem for a second order elliptic equation in a general convex domain.* Lecture Notes in Math 564, Springer-Verlag (1976), 135–151.
[6] M. LEWICKA AND P.B. MUCHA, *A local and global well-posedness results for the general stress-assisted diffusion systems.* J. Elast. 123 (2016), no. 1, 19–41.
[7] C. TRUESDELL, *Mechanical basis of diffusion.* J. Chem. Phys. 37 (1962), 2336–2344.

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VIRTUAL ELEMENT ANALYSIS FOR THE BUCKLING PROBLEM OF THIN PLATES

DAVID MORA AND IVÁN VELÁSQUEZ

Abstract. The aim of this work is to develop a virtual element method (VEM) for the buckling problem for thin plates on polygonal meshes. A variational formulation based on the Kirchhoff model depending on the transverse displacement of the plate is written. The classical approximation theory for compact operators is employed in order to establish the optimal order error estimates for the eigenfunctions and a double order for the eigenvalues. Finally, we report some numerical experiments illustrating the behaviour of the proposed scheme and confirming our theoretical results on different families of meshes.

References

[1] P. F. Antonietti, L. Beirão da Veiga, S. Scacchi and M. Verani, *A C^1 virtual element method for the Cahn–Hilliard equation with polygonal meshes*. SIAM J. Numer. Anal., 54 (2016), no. 1 , pp. 36-56.
[2] F. Brezzi and L.D. Marini, *Virtual elements for plate bending problems*. Comput. Methods Appl. Mech. Engrg., 253 (2013), pp. 455–462.
[3] D. Mora, G. Rivera and I. Velásquez, *A virtual element method for the vibration problem of Kirchhoff plates*. ESAIM Math. Model. Numer. Anal., to appear.
[4] D. Mora and R. Rodríguez, *A piecewise linear finite element method for the buckling and the vibration problems of thin plates*. Math. Comp., 78 (2009), no. 268 , pp. 1891-1917.

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ANALYSIS OF A CONSERVATIVE MIXED-FEM FOR THE STATIONARY NAVIER–STOKES PROBLEM

JESSIKA CAMAÑO, CARLOS GARCÍA, AND RICARDO OYARZÚA

Abstract. In this work we propose and analyze a new conservative mixed finite element method for the Navier–Stokes problem posed in non-standard Banach spaces. Our approach is based on the introduction of a pseudostress tensor relating the velocity gradient with the convective term, leading to a mixed formulation where the aforementioned pseudostress tensor and the velocity are the main unknowns of the system. Then the associated Galerkin scheme can be defined by employing Raviart–Thomas elements of degree k for the pseudostress tensor and discontinuous piecewise polynomial elements of degree k for the velocity. With this choice of spaces, the equilibrium equation is exactly satisfied if the external force belongs to the velocity discrete space, thus the method is conservative, which constitutes one of the main feature of our approach. For both, the continuous and discrete problems, the Banach–Nečas–Babuška and Banach's fixed point theorems are employed to prove unique solvability. We also provide the convergence analysis and particularly prove that the error decay with optimal rate of convergence. Further variables of interest, such as the fluid pressure, the fluid vorticity and the fluid velocity gradient, can be easily approximated as a simple postprocess of the finite element solutions with the same rate of convergence. Finally, several numerical results illustrating the performance of the method are provided.

References

[1] J. Camaño, R. Oyarzúa and G. Tierra, *Analysis of an augmented mixed-fem for the Navier–Stokes problem*. Math. Comp. 86 (2017), no. 304, 589-615.
[2] J. Camaño, C. Muñoz and R. Oyarzúa, *Numerical analysis of a dual-mixed problem in non-standard Banach spaces*. Electron. Trans. Numer. Anal. 48 (2018), 114-130.
[3] J. Camaño, C. García and R. Oyarzúa, *Analysis of a conservative mixed-FEM for the stationary Navier–Stokes problem*, Preprint 2018-25, Centro de Investigacion en Ingenieria Matematica (CI²MA), Universidad de Concepcion, Chile, (2018).

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14th CI^{2}MA FOCUS SEMINAR: Finite Element and Related Methods for PDEs in Continuum Mechanics
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NUMERICAL ANALYSIS OF A TIME-DOMAIN ELASTOACOUSTIC PROBLEM

RODOLFO ARAYA, RODOLFO RODRÍGUEZ, AND PABLO VENEGAS

Abstract. This work deals with the numerical analysis of a system of second-order in time partial differential equations modeling the vibrations of a coupled system that consists of an elastic solid in contact with an inviscid compressible fluid. We analyze a weak formulation with the unknowns in both media being the respective displacement fields. For its numerical approximation, we propose first a semi-discrete in space discretization based on standard Lagrangian elements in the solid and Raviart-Thomas elements in the fluid. We establish its wellposedness and derive error estimates in appropriate norms for the proposed scheme. In particular, we obtain an $\mathrm{L}^\infty(\mathrm{L}^2)$ optimal rate of convergence under minimal regularity assumptions of the solution, which are proved to hold for appropriate data of the problem. Then, we consider a fully discrete approximation based on a family of implicit finite difference schemes in time, from which we obtain optimal error estimates for sufficiently smooth solutions. Finally, we report some numerical results, which allow us to assess the performance of the method. These results also show that the numerical solution is not polluted by spurious modes as is the case with other alternative approaches.

References

[1] Basson, M. and van Rensburg, N. F. J., *Galerkin finite element approximation of general linear second order hyperbolic equations.* Numer. Func. Anal. Opt. (2013), no. 34, 976-1000.
[2] Bermúdez, A. and Durán, R. and Muschietti, M. A. and Rodríguez, R. and Solomin, J., *Finite element vibration analysis of fluid-solid systems without spurious modes.* SIAM J. Numer. Anal. (1995), no. 32, 1280-1295.
[3] A. Bermúdez and R. Rodríguez and D. Santamarina, *Finite element approximation of a displacement formulation for time-domain elastoacoustic vibrations.* J. Comput. Appl. Math. (2003), no. 152, 17-34.
[4] C. García and G. N. Gatica and S. Meddahi and A. Márquez, *A fully discrete scheme for the pressure-stress formulation of the time-domain fluid-structure interaction problem.* Calcolo (2017), no. 54, 1419-1439.
[5] Karaa, S, *Finite element θ-schemes for the acoustic wave equation.* Adv. Appl. Math. Mech. (2011), no. 3, 181-203.

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DG METHODS FOR SPECTRAL PROBLEMS

FELIPE LEPE, SALIM MEDDAHI, DAVID MORA, AND RODOLFO RODRÍGUEZ

ABSTRACT. The discontinuous Galerkin method (DG) has been studied in several problems of fluid and solid mechanics. In particular, the interest of consider the DG method for eigenvalue problems appears in [1] for the Laplacian operator, where the spectral theory for noncompact operators [4, 5] is adapted. This paper has led to other challenging problems like the Maxwell's eigenvalue problem analyzed in [2, 3]. In this talk we present results for an interior penalty DG method for a mixed formulation of the elasticity eigenvalue problem like the studied in [6] and a stress formulation of the Stokes eigenvalue problem, where the pseudostress tensor is incorporated in order to eliminate the velocity and the pressure fields. For both problems we present a spectral characterization and results related to the non pollution of the spectrum. We present numerical results to asses the performance of the methods in each case.

REFERENCES

[1] P. F. ANTONIETTI, A. BUFFA AND I. PERUGIA, *Discontinuous Galerkin approximation of the Laplace eigenproblem*, Comput. Methods Appl. Mech. Engrg., 195 (2006), 3483–3503.
[2] A. BUFFA, P. HOUSTON AND I. PERUGIA, *Discontinuous Galerkin computation of the Maxwell eigenvalues on simplicial meshes,* J. Comput. Appl. Math. 2014 (2007), 317–333.
[3] A. BUFFA AND I. PERUGIA, *Discontinuous Galerkin approximation of the Maxwell eigenproblem*, SIAM J. Numer. Anal., 44 (2006), 2198–2226.
[4] J. DESCLOUX, N. NASSIF, AND J. RAPPAZ, *On spectral approximation. Part 1: The problem of convergence*, RAIRO Anal. Numér., 12 (1978), 97–112.
[5] J. DESCLOUX, N. NASSIF, AND J. RAPPAZ, *On spectral approximation. Part 2: Error estimates for the Galerkin method*, RAIRO Anal. Numér., 12 (1978), 113–119.
[6] S. MEDDAHI, D. MORA, AND R. RODRÍGUEZ, *Finite element spectral analysis for the mixed formulation of the elasticity equations*, SIAM J. Numer. Anal., 51 (2013), 1041–1063.
[7] S. MEDDAHI, D. MORA, AND R. RODRÍGUEZ, *A finite element analysis of a pseudostress formulation for the Stokes eigenvalue problem*, IMA J. Numer. Anal., 35 (2015), 749–766.

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AN ASYMPTOTIC MODEL BASED ON MATCHING FAR AND NEAR FIELDS FOR THIN GRATINGS PROBLEMS

PETER B. MONK, CINTHYA RIVAS, RODOLFO RODRÍGUEZ, AND MANUEL E. SOLANO

Abstract. In this work, we have devised an asymptotic model for calculating electromagnetic diffraction and absorption in planar multilayered structures having a shallow surface-relief grating. Far from the grating we assume that the solution can be written as a power series in terms of the grating thickness δ. The coefficients of this expansion are smooth up to the grating. However, the expansion approximates the solution only sufficiently far from the grating (far field approximation). Near the grating, we assume that there exists another expansion in powers of δ. Moreover, there is an overlapping domain, where both expansion are valid. The method based is on matching these two expansions on a thin overlapping zone. By so doing, we obtain explicitly the equations satisfied by the first terms in the power series, by truncating the δ^2 or higher order terms.

References

[1] B. Delourme, H. Haddar, and P. Joly, *Approximate models for wave propagation across thin periodic interfaces*, J. Math. Pures Appl. 98 (2012), 28–71.
[2] E. G. Loewen and E. Popov, *Diffraction Gratings and Applications* Marcel Dekker, (1997).
[3] M. E. Solano, M. Faryad, A. Lakhtakia, and P. B. Monk, *Comparison of rigorous coupled-wave approach and finite element method for photovoltaic devices with periodically corrugated metallic back reflector*, J. Opt. Soc. Am. A 31 (2014), 2275–2284.
[4] C. Rivas, Manuel E. Solano, R. Rodriguez, P. B. Monk and A. Lakhtakia, *Asymptotic model for finite-element calculations of diffraction by shallow metallic surface-relief gratings*, J. Opt. Soc. Am. A 34 (2017), 68–79.

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A MIXED VIRTUAL ELEMENT METHOD FOR THE NAVIER-STOKES EQUATIONS

GABRIEL N. GATICA, MAURICIO MUNAR, AND FILANDER SEQUEIRA

Abstract. A mixed virtual element method (mixed-VEM) for a pseudostress-velocity formulation of the two-dimensional Navier-Stokes equations with Dirichlet boundary conditions is proposed and analyzed in this work. More precisely, we employ a dual-mixed approach based on the introduction of a nonlinear pseudostress linking the usual linear one for the Stokes equations and the convective term. In this way, the aforementioned new tensor together with the velocity constitute the only unknowns of the problem, whereas the pressure is computed via a postprocessing formula. In addition, the resulting continuous scheme is augmented with Galerkin type terms arising from the constitutive and equilibrium equations, and the Dirichlet boundary condition, all them multiplied by suitable stabilization parameters, so that the Banach fixed-point and Lax-Milgram theorems are applied to conclude the well-posedness of the continuous and discrete formulations. Next, we describe the main VEM ingredients that are required for our discrete analysis, which, besides projectors commonly utilized for related models, include, as the main novelty, the simultaneous use of virtual element subspaces for $\mathbf{H}^1$ and $\mathbb{H}(\mathbf{div})$ in order to approximate the velocity and the pseudostress, respectively. Then, the discrete bilinear and trilinear forms involved, their main properties and the associated mixed virtual scheme are defined, and the corresponding solvability analysis is performed using again appropriate fixed-point arguments. Moreover, Strang-type estimates are applied to derive the *a priori* error estimates for the two components of the virtual element solution as well as for the fully computable projections of them and the postprocessed pressure. As a consequence, the corresponding rates of convergence are also established. Finally, we follow the same approach employed in previous works by some of the authors and introduce an element-by-element postprocessing formula for the fully computable pseudostress, thus yielding an optimally convergent approximation of this unknown with respect to the broken $\mathbb{H}(\mathbf{div})$-norm.

References

[1] B. Ahmad, A. Alsaedi, F. Brezzi, L.D. Marini and A. Russo, *Equivalent projectors for virtual element methods*. Comput. Math. Appl. 66 (2013), no. 3, 376–391.

This research was partially supported by CONICYT-Chile through the project AFB170001 of the PIA Program: Concurso Apoyo a Centros Cientificos y Tecnologicos de Excelencia con Financiamiento Basal, and the Becas-CONICYT Programme for foreign students; by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción; and by Universidad Nacional (Costa Rica), through the project 0106-16.

[2] E. Cáceres and G.N. Gatica, *A mixed virtual element method for the pseudostress-velocity formulation of the Stokes problem.* IMA J. Numer. Anal. 37 (2017), no. 1, 296–331.

[3] E. Cáceres, G.N. Gatica and F.A. Sequeira, *A mixed virtual element method for the Brinkman problem.* Math. Models Methods Appl. Sci. 27 (2017), no. 4, 707–743.

[4] E. Cáceres, G.N. Gatica and F.A. Sequeira, *A mixed virtual element method for quasi-Newtonian Stokes flows.* SIAM J. Numer. Anal. 56 (2018), no. 1, 317–343.

[5] G.N. Gatica, M. Munar and F.A. Sequeira, *A mixed virtual element method for the Navier-Stokes equations.* Preprint 2017-23, Centro de Investigación en Ingeniería Matemática, Universidad de Concepción, Chile [available at: `https://www.ci2ma.udec.cl/publicaciones/prepublicaciones`]

[6] G.N. Gatica, M. Munar and F.A. Sequeira, *A mixed virtual element method for a nonlinear Brinkman model of porous media flow.* Calcolo 55 (2018), no. 2, article:21.

[7] F. Brezzi, R.S. Falk and L.D. Marini, *Basic principles of mixed virtual element methods.* ESAIM Math. Model. Numer. Anal. 48 (2014), no. 4, 1227–1240.

[8] J. Camaño, G.N. Gatica, R. Oyarzúa and G. Tierra, *An augmented mixed finite element method for the Navier-Stokes equations with variable viscosity.* SIAM J. Numer. Anal. 54 (2016), no. 2, 1069–1092.

[9] J. Camaño, R. Oyarzúa and G. Tierra, *Analysis of an augmented mixed-FEM for the Navier-Stokes problem.* Math. Comp. 86 (2017), no. 304, 589–615.

[10] L. Beirão da Veiga, F. Brezzi, L.D. Marini and A. Russo, *Mixed virtual element methods for general second order elliptic problems on polygonal meshes.* ESAIM Math. Model. Numer. Anal. 50 (2016), no. 3, 727–747.

[11] L. Beirão da Veiga, C. Lovadina and G. Vacca, *Virtual elements for the Navier-Stokes problem on polygonal meshes.* SIAM J. Numer. Anal. 56 (2018), no. 3, 1210–1242.

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ON EXPONENTIAL STABILITY FOR THERMOELASTIC PLATES: COMPARISON AND SINGULAR LIMITS

JAIME MUÑOZ RIVERA, REINHARD RACKE, AND MAURICIO SEPÚLVEDA

Abstract. We consider different models of thermoelastic plates in a bounded reference configuration: with Fourier heat conduction or with the Cattaneo model, and with or without inertial term. Some models exhibit exponential stability, others are not exponential stable. In the cases of exponential stability, we give an explicit estimate for the rate of decay in terms of the essential parameters appearing (delay $\tau \geq 0$, inertial constant $\mu \geq 0$). This is first done using multiplier methods directly in L^2-spaces, then, second, with eigenfunction expansions imitating Fourier transform techniques used for related Cauchy problems. The explicit estimates allow for a comparison. The singular limits $\tau \to 0$, and $\mu \to 0$ are also investigated in order to understand the mutual relevance for the (non-) exponential stability of the models. Numerical simulations underline the results obtained analytically, and exhibit interesting coincidences of analytical and numerical estimates, respectively.

References

[1] Fernández Sare, Hugo D.; Muñoz Rivera, Jaime E. *Optimal rates of decay in 2-d thermoelasticity with second sound.* J. Math. Phys. 53 (2012), no. 7, 073509, 13 pp.
[2] Fernández Sare, Hugo D.; Racke, Reinhard *On the stability of damped Timoshenko systems: Cattaneo versus Fourier law.* Arch. Ration. Mech. Anal. 194 (2009), no. 1, 221-251.
[3] Muñoz Rivera, Jaime E.; Racke, Reinhard *Smoothing properties, decay, and global existence of solutions to nonlinear coupled systems of thermoelastic type.* SIAM J. Math. Anal. 26 (1995), no. 6, 1547-1563.
[4] Muñoz Rivera, Jaime E.; Racke, Reinhard *Large solutions and smoothing properties for nonlinear thermoelastic systems.* J. Differential Equations 127 (1996), no. 2, 454-483.
[5] Muñoz Rivera, Jaime E.; Racke, Reinhard; Sepúlveda, Mauricio; Vera Villagrán, Octavio *On exponential stability for thermoelastic plates: comparison and singular limits*, Konstanzer Schriften Math. 374 (2018).

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A POSTERIORI ERROR ANALYSIS OF HDG METHODS IN FLUID MECHANICS

RODOLFO ARAYA, MANUEL SOLANO, AND PATRICK VEGA

Abstract. We introduce an analyze *a posteriori* error estimators, of the residual type, for a hybridizable discontinuous Galerkin (HDG) method applied to two problems arising from fluid mechanics. The first one is a gradient-velocity-pressure formulation of the Brinkman problem, where, in order to derive our estimator, we use the Oswald interpolant and a suitable constructed postprocessed approximation of the velocity. The second problem is a gradient-velocity-pressure formulation of Oseen equations. In this case, in addition to the properties of the Oswald interpolant, we employ a weighted function technique to control the L^2-error of the velocity. For both cases we establish reliability and local efficiency of the estimator for the L^2-error of the velocity gradient and the pressure and the H^1-error of the velocity, with constants written explicitly in terms of the physical parameters. Numerical experiments validate the quality of the adaptive scheme.

References

[1] R. Araya, M. Solano and P. Vega, *Analysis of an adaptive HDG method for the Brinkman problem*, IMA J. Numer. Anal., accepted.

[2] R. Araya, M. Solano and P. Vega, *A posteriori error analysis of an HDG method for the Oseen problem*, in preparation.

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ANALYSIS OF A MIXED FEM FOR STATIONARY INCOMPRESSIBLE MAGNETO-HYDRODYNAMICS

JESSIKA CAMAÑO, CARLOS GARCÍA, AND RICARDO OYARZÚA

Abstract. In this paper we propose and analyze a new mixed finite element method for stationary incompressible magneto-hydrodynamics posed in non-standard Banach spaces. Our approach is based on the introduction of a pseudostress tensor relating the velocity gradient with the convective term, leading to a mixed formulation where the aforementioned pseudostress tensor and the velocity as the main hydrodynamic unknowns, with magnetic field and a Lagrange multiplier as magnetic unknowns. Then the associated Galerkin scheme can be defined by employing Raviart–Thomas elements of degree k for the pseudostress tensor, discontinuous piecewise polynomial elements of degree k for the velocity, Nédélec elements of degree k for the magnetic field and discontinuous piecewise polynomial elements of degree k for the respective Lagrange multiplier. For both, the continuous and discrete problems, the Banach–Nečas–Babuška and Banach's fixed point theorems are employed to prove unique solvability. We also provide the convergence analysis and particularly prove that the error decay with optimal rate of convergence. Further variables of interest, such as the fluid pressure, the fluid vorticity and the fluid velocity gradient, can be easily approximated as a simple postprocess of the finite element solutions with the same rate of convergence.

References

[1] J. Camaño, C.García and R. Oyarzúa, *Analysis of a conservative mixed-FEM for the stationary Navier-Stokes problem.* Preprint 2018-25, Centro de Investigación en Ingeniería Matemática (CI^{2}MA), UDEC, (2018).
[2] J. Camaño, R. Oyarzúa and G. Tierra, *Analysis of an augmented mixed-fem for the Navier-Stokes problem.* Math. Comp. 86 (2017), no. 304, 589615.
[3] J. Camaño, C. Muñoz and R. Oyarzúa, *Numerical analysis of a dual-mixed problem in non-standard Banach spaces.* ETNA - Electronic Transactions on Numerical Analysis 48 (2018), pp. 114–130.
[4] Dominik Schötzau, *Mixed finite element methods for stationary incompressible magneto-hydrodynamics.* Numer. Math. 96 (2004), no. 4, pp. 771–800.
[5] Chen Greif, Dan Li, Dominik Schötzau, Xiaoxi Wei, *A mixed finite element method with exactly divergence-free velocities for incompressible magnetohydrodynamics.* Comput. Methods Appl. Mech. Engrg. 199 (2010), no. 45-48, pp. 2840–2855.

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[6] P. Houston, D. Schötzau, X. Wei, *A mixed DG method for linearized incompressible magnetohydrodynamics*. J. Sci. Comp. 40 (2009), no. 1-3, pp. 281–314.
[7] J.C. Nédélec, *A new family of mixed finite elements in* $\mathbb{R}^3$. Numer. Math, 50 (1986), no. 1, pp. 57–81.

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ON THE COUPLING OF VEM AND BEM IN TWO AND THREE DIMENSIONS

GABRIEL N. GATICA AND SALIM MEDDAHI

Abstract. This paper introduces and analyzes the combined use of the virtual element method (VEM) and the boundary element method (BEM) to numerically solve linear transmission problems in 2D and 3D. As a model we consider an elliptic equation in divergence form holding in an annular domain coupled with the Laplace equation in the corresponding unbounded exterior region, together with transmission conditions on the interface and a suitable radiation condition at infinity. We employ the usual primal formulation in the bounded region, and combine it, by means of the Costabel & Han approach, with the boundary integral equation method in the exterior domain. As a consequence, and besides the original unknown of the model, its normal derivative in 2D, and both its normal derivative and its trace in the 3D case, are introduced as auxiliary non-virtual unknowns. Moreover, for the latter case, a new and more suitable variational formulation for the coupling is introduced. In turn, the main ingredients required by the discrete analyses include the virtual element subspaces for the domain unknowns, explicit polynomial subspaces for the boundary unknowns, and suitable projection and interpolation operators that allow to define the corresponding discrete bilinear forms. In particular, two VEM/BEM schemes are proposed in the three-dimensional case, one of them mimicking the non-symmetric interior penalty discontinuous Galerkin method. Then, as for the continuous formulations, the classical Lax-Milgram lemma is employed to derive the well-posedness of our coupled VEM-BEM scheme. Finally, a priori error estimates in the energy and weaker norms, and corresponding rates of convergence for the solution as well as for a fully computable projection of the virtual component of it, are provided.

References

[1] M. Aurada, M. Feischl, T. Führer, M. Karkulik, J.M. Melenk and D. Praetorius, *Classical FEM-BEM coupling methods: nonlinearities, well-posedness, and adaptivity.* Comput. Mech. 51 (2013), no. 4, 399-419.
[2] L. Beirão da Veiga, F. Brezzi, A. Cangiani, L.D Marini, G. Manzini and A. Russo, *Basic principles of virtual elements methods.* Math. Models Methods Appl. Sci. 23 (2013), no. 1, 199–214.

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[3] L. Beirão da Veiga, F. Brezzi, L.D. Marini and A. Russo, *Virtual element method for general second-order elliptic problems on polygonal meshes.* Math. Models Methods Appl. Sci. 26 (2016), no. 4, 729–750.

[4] S.C. Brenner, Q. Guan and L.Y. Sung, *Some estimates for virtual element methods.* Comput. Methods Appl. Math. 17 (2017), no. 4, 553–574.

[5] F. Brezzi and C. Johnson, *On the coupling of boundary integral and finite element methods.* Calcolo 16 (1979), no. 2, 189–201.

[6] E. Cáceres and G.N. Gatica, *A mixed virtual element method for the pseudostress-velocity formulation of the Stokes problem.* IMA J. Numer. Anal. 37 (2017), no. 1, 296–331.

[7] M. Costabel, *Symmetric methods for the coupling of finite elements and boundary elements.* In: Boundary Elements IX (C.A. Brebbia, G. Kuhn, W.L. Wendland eds.), Springer, Berlin, pp. 411-420, (1987).

[8] G.N. Gatica, G.C. Hsiao, S. Meddahi and F.J. Sayas, *New developments on the coupling of mixed-FEM and BEM for the three-dimensional exterior Stokes problem.* Int. J. Numer. Anal. Model. 13 (2016), no. 3, 457–492.

[9] G.N. Gatica, M. Munar and F.A. Sequeira, *A mixed virtual element method for a nonlinear Brinkman model of porous media flow.* Calcolo 55 (2018), no. 2, article:21.

[10] H. Han, *A new class of variational formulations for the coupling of finite and boundary element methods.* J. Comput. Math. 8 (1990), no. 3, 223–232.

[11] C. Johnson and J.C. Nédélec, *On the coupling of boundary integral and finite element methods.* Math. Comp. 35 (1980), no. 152, 1063–1079.

[12] S. Meddahi, J. Valdés, O. Menéndez and P. Pérez, *On the coupling of boundary integral and mixed finite element methods.* J. Comput. Appl. Math. 69 (1996), no. 1, 113–124.

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