Centro de Investigación en Ingeniería Matemática

Universidad de Concepción

5th CI^{2}MA Focus Seminar
"Numerical methods for multiphase flows, geophysics, and related problems"
Supported by Conicyt Project Anillo ACT1118 (ANANUM)

July 10, 2013
Auditorio Alamiro Robledo
Facultad de Ciencias Físicas y Matemáticas
Universidad de Concepción

Organizer: Raimund Bürger

Programme

15.00 **Cristóbal Castro** (U. de Tarapacá, Arica, Chile):
Multiphase flow models applied to three dimensional inundation processes

15.30 **Amélie Rambaud** (Universidad del Bío-Bío):
A general framework for the derivation of turbulent shallow water models

16.00 **Pep Mulet** (Universidad de Valencia, España):
Adaptive refinement techniques for WENO schemes applied to a quasi-relativistic Vlasov-Maxwell model for laser-plasma interaction

16.30 **Christian Ihle** (AMTC, Universidad de Chile):
Challenges of ore concentrate slurry pipeline transport

17.00 **Coffee break**

17.30 **Héctor Torres** (Depto. de Matemáticas, Universidad de La Serena):
Numerical methods for a coupled flow-transport system modeling sedimentation processes

18.00 **Fernando Betancourt** (Depto. de Ingeniería Metalúrgica, U. de Concepción):
Continuous sedimentation of suspensions with time-varying feed properties

18.30 **Raimund Bürger** (CI^{2}MA & Depto. de Ing. Matemática, U. de Concepción):
On systems of conservation laws modeling the settling of polydisperse suspensions

19.00 **Luis M. Villada** (CI^{2}MA & Depto. de Ing. Matemática, U. de Concepción):
IMEX-Runge-Kutta methods applied to polydisperse sedimentation with compression

20.30 **Seminar Dinner**

Practical information

Seminar participants who would like to join dinner should register with CI^{2}MA secretary:

Ms Angelina Fritz, CI^{2}MA
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Abstracts

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MULTIPHASE FLOW MODELS APPLIED TO THREE DIMENSIONAL INUNDATION PROCESSES

CRISTÓBAL E. CASTRO

Abstract. In this presentation we discuss a new approach for simulating three-dimensional inundation process where multiphase models can be used to better represent the real dynamics of such geophysical problems.

We have seen in resent years the devastating consequences of tsunami events in Chile and Japan. It is the standard today for tsunami warning centres the use of depth-average models to produce inundation maps. While these tools are extremely important for local authorities, there are still some question marks about the validity of such models to truly represent the real dynamics of the process. In particular when we are interested in overtopping and breaking waves and the interaction between fluid and solid structures.

We discuss recent publications [4, 2, 3] where starting from the general multiphase-flow model of Baer and Nunziato [1] (BN) a two-fluid (liquid-air) model is adapted and tested to simulate free surface flows with promising results.

In these models there are some assumptions that allows to simplify the BN system. These assumptions reduce the computational cost while keeping the three-dimensionality description of the liquid fluid .

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A GENERAL FRAMEWORK FOR THE DERIVATION OF TURBULENT SHALLOW WATER MODELS

AMÉLIE RAMBAUD AND JEAN-PAUL VILA

ABSTRACT. In view of applying our process to bilayer, and multilayer flows, we present here a systematic and general methodology to derive two moments approximate models of shallow water type for a single thin film down an incline in a turbulent regime. Some recent related works are [4, 5]. If the laminar case is now well understood and rigorously justified, few turbulent models exist. We consider a model of Mixing Length law [3] to describe turbulence. We perform a long wave asymptotic of the Navier-Stokes equations and construct a Hilbert expansion up to the second order with respect to the small aspect ratio (or film parameter) ε of the solution. This expansion appears to be singular because of the turbulence model : two distinct behaviors of the fluid are identifyied, the so-called external and internal expansions of the solution (we refer to [1] for the vocabulary about singular expansions) . The derivation of Shallow Water models from the previous expansions is then classical, but furnishes here new corrective terms of consistency. In particular they differ from more generally used SW systems with friction [2]. We will then briefly present the work plan to deal with multilayer flows.

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ADAPTIVE REFINEMENT TECHNIQUES FOR WENO SCHEMES APPLIED TO A QUASI-RELATIVISTIC VLASOV-MAXWELL MODEL FOR LASER-PLASMA INTERACTION

SIMON LABRUNIE, PEP MULET, AND FRANCESCO VECIL

Abstract. In this work we deal with the simulation of plasma heating by a laser pulse.The most complete model for carrying out this simulation is the full 3D Vlasov-Maxwell equations, which are 6D in phase-space (space, momenta). Since this is mostly unbearable for numerical codes, in our setting this simulation is governed by a one-dimensional Vlasov-Maxwell model [1] (2D in phase-space) for the probability density of ions in the plasma. A quasi-relativistic approximation for the Lorentz factor enables the use of time splitting schemes [3], a technique that may result in faster and simpler schemes.

Since strong gradients and filaments at certain regions in phase space may appear, we explore different WENO-based schemes for the Vlasov and Maxwell parts and use an adaptive mesh refinement (AMR) strategy, similar to the one proposed in [2], to save computational resources. Preliminary results for the efficiency of the adaptive scheme are shown.

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CHALLENGES OF ORE CONCENTRATE SLURRY PIPELINE TRANSPORT

CHRISTIAN IHLE

Abstract. Due to their cost-effectiveness, long distance slurry pipelines are a widespread means of transporting ore from processing plants to port destinations. Water scarcity in north Chile along with the relatively recent growth of energy costs imply optimal operational regimes that depart from traditionally specified flows and concentrations [1]. In particular, it is required to transport the ore at higher flow rates and concentrations than in typical design specifications. This new reality pose a challenge to new transport systems, which not only require to operate at steady state, but also need to allow for potentially prolonged idle times in optimal operational regimes. When stopping an otherwise flowing concentrate line, the segregation of the initially homogeneous slurry starts, with diverse solid and liquid flow components along the pipeline axis due to the coupled effect of gravity and local hydrodynamic effects such as the Boycott effect [2] and hydrodynamic segregation [4, 5]. This complex process may evolve to the formation of various solid plugs which may significantly hinder the subsequent system startup, depending on several variables, including system geometry, slurry concentration and particle size distribution. In this presentation, a numerical and experimental approach to this problem is posed, both with emphasis on the solid matter flow and distribution and on the potential effect of particle polydispersity on the sedimentation patterns.

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NUMERICAL METHODS FOR A COUPLED FLOW-TRANSPORT SYSTEM MODELING SEDIMENTATION PROCESSES

HECTOR TORRES

Abstract. A model of sedimentation processes is given by hyperbolic (parabolic) equation describing the evolution of the solid concentration coupled with the Navier-Stokes equations for an incompressible fluid describing the motion of the mixture. An adaptive multiresolution scheme is proposed for the numerical solution of a two-dimensional model, describing the batch sedimentation in an inclined closed vessel [1]. We consider a finite volume element scheme [2] for numerical solutions in different configurations for the sedimentation process. Some numerical experiments illustrate properties of the model and the satisfactory performance of the proposed methods in two and three spatial dimensions.

This contribution is a collaboration with R. Bürger (Concepción), K. Schneider (Marseille, France) and R. Ruiz-Baier (Lausanne, Switzerland).

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CONTINUOUS SEDIMENTATION OF SUSPENSIONS WITH TIME-VARYING FEED PROPERTIES

FERNANDO BETANCOURT, RAIMUND BÜRGER, STEFAN DIEHL, AND SEBASTIAN FARÅS

ABSTRACT. Most of the available models for the design, simulation and control of clarifier-thickener (CT) units for gravity solid-liquid separation in mineral processing are based on a single spatially one-dimensional nonlinear conservation partial differential equation (PDE) of convection-diffusion type for the solids concentration as a function of depth and time. We extend a common CT model of this kind to include suspensions with time-varying feed properties by considering the Stokes velocity of an individual solid particle as a time-dependent function resulting from changes in the state of flocculation, which is expressed by a new scalar field variable, whose evolution within the CT unit is governed by an additional conservation PDE. Hence, the new model consists of two coupled nonlinear PDEs. A numerical scheme is suggested, numerical solutions are presented and the capability of the model to describe CT control is investigated.

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ON SYSTEMS OF CONSERVATION LAWS MODELING THE SETTLING OF POLYDISPERSE SUSPENSIONS

RAIMUND BÜRGER

ABSTRACT. It is the purpose of this contribution to provide an overview on advances in the mathematical analysis and numerical solution of systems of conservation laws motivated by models of sedimentation of polydisperse suspensions of small solid particles dispersed in a viscous fluid. It is assumed that particles belong to a number N of species differing in size or density, which gives rise to a strongly coupled system of N scalar, nonlinear first-order partial differential equations. After a brief introduction to typical applications (mainly in mineral processing, chemical engineering and volcanology), the basic model assumptions, and the model equations in final form [1], several mathematical topics are discussed, starting with the problem of proving hyperbolicity of the system of conservation laws along with the related issue of suspension stability [2]. We then outline how the hyperbolicity analysis provides the ingredients necessary for the implementation of high-resolution numerical schemes [3]. Finally we deal with the numerical solution of extensions of the governing equations, namely of a system with additional degenerate diffusion terms [4] modeling sediment compressibility and of a multilayer shallow water model with horizontal flow [5].

This contribution is based on recent joint work with R. Donat and P. Mulet (Valencia, Spain), E.D. Fernández-Nieto and E.H. Koné (Sevilla, Spain), T. Morales de Luna (Córdoba, Spain), L.M. Villada (Concepción) and C.A. Vega (Barranquilla, Colombia).

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IMEX-RUNGE-KUTTA METHODS APPLIED TO POLYDISPERSE SEDIMENTATION WITH COMPRESSION

RAIMUND BÜRGER, PEP MULET, AND LUIS M. VILLADA

Abstract. Implicit-explicit (IMEX) methods are suitable for the solution of nonlinear convection-diffusion equations [2, 4, 6], since the stability restrictions, coming from the explicitly treated convective part, are much less severe than those that would be deduced from an explicit treatment of the diffusive term [3]. These schemes usually combine an explicit Runge-Kutta scheme for the time integration of the convective part with a diagonally implicit one for the diffusive part. The application of these schemes to multi-species kinematic flow models with strongly degenerate diffusive corrections requires the solution of highly nonlinear and non-smooth systems of algebraic equations. Since the efficient solution of these systems by the Newton-Raphson method requires some degree of smoothness, it is proposed to regularize the diffusion coefficients in the model and to apply suitable techniques to solve these nonlinear systems in an efficient way. Numerical examples arising from models of polydisperse sedimentation [3] confirm the efficiency of the methods proposed.

References

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