On constitutive functions for hindered settling velocity in 1-D settler models: selection of appropriate model structure

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Abstract

Advanced 1-D models for Secondary Settling Tanks (SSTs) aim to explicitly account for several phenomena that influence the settling process (such as hindered settling and compression settling). For each of these phenomena a valid mathematical expression needs to be selected and its parameters calibrated to obtain a model that can be used for operation and control. The presented work evaluates several available expressions for hindered settling based on long-term batch settling data. The analysis shows that the exponential forms which are most commonly used in traditional SST models not only account for hindered settling but partly lump other phenomena (compression) as well. This makes them unsuitable for advanced 1-D models that explicitly include each phenomenon in a modular way. A power-law function is shown to be more appropriate to describe the hindered settling velocity in advanced 1-D SST models.

1 Introduction

Settling tank models are valuable tools to improve understanding of the underlying mechanisms and processes affecting SST performance. Consequently, these models can serve as a tool for process optimisation and control. The modelling of SSTs requires a proper mathematical description of the settling behaviour of Activated Sludge (AS). However, the development of a general method to characterise the complete settling process is quite challenging due to the simultaneous occurrence of distinct settling regimes (i.e. discrete, hindered and compression settling) at different depths of the settler.
Discrete settling occurs at dilute sludge concentrations where the settling behaviour is governed by each particle's individual properties (i.e. Stokesian settling). As individual particle dynamics are cumbersome to model, discrete settling is traditionally incorporated into the hindered settling regime [Takács et al., 1991]. In the hindered settling regime, all

particles are assumed to settle as a zone with a constant settling velocity. The hindered settling velocity is thus a function of the total sludge concentration with higher concentrations resulting in lower settling velocities. Several empirical expressions describe this decreasing relation between the settling velocity (v_{hs}) and the solids concentration (X) [Vesilind, 1968, Cole, 1968, Richardson and Zaki, 1954, Takács et al., 1991, Cho et al., 1993, Watts et al., 1996, Lakehal and Krebs, 1999, Zhang et al., 2016].

At high solids concentrations, previously settled solids will thicken under the weight of overlying particles and undergo compression. The particles form a compressible network resulting in an additional force which slows down the hindered settling velocity. The dynamics of the settling velocity in the compression region will thus differ from that in the hindered settling region. Several mathematical expressions to describe the settling velocity in the compression region (v_{comp}) have been developed ranging from basic extensions of the hindered settling velocity [Zhang et al., 2006, Stricker et al., 2007] to more fundamentally supported equations [Cacossa and Vaccari, 1994, Kinnear, 2002, De Clercq et al., 2008, Ramin et al., 2014].

Traditional SST models such as the commonly used Takács model [Stenstrom, 1976, Vitasovic, 1986, Takács et al., 1991] consider only hindered settling and apply a coarse discretisation to solve the underlying Partial Differential Equation (PDE). These type of models are used in many simulation programs and perform reasonably well under normal dry weather conditions. However, their predictions lose realism under situations that diverge from normal operating conditions (e.g. peak flows due to rain events) [Jeppsson and Diehl, 1996, Plósz et al., 2011, Bürger et al., 2012, Torfs et al., 2015]. Under such conditions the inclusion of compressive settling in the mathematical description of the settling behaviour will have a strong influence on the estimation of the Sludge Blanket Height (SBH) and underflow concentration which are two important variables for control and operation of the clarifier [Torfs et al., 2015]. Hence, appropriate predictions of SST performance under peak flow conditions in terms of underflow and SBH require a more sophisticated model including compression settling.

Recent advances in 1-D models have focussed on two issues. On the one hand numerical schemes have been improved to solve the underlying PDE [Plósz et al., 2011, Bürger et al., 2012, Li and Stenstrom, 2014] and on the other hand this PDE has been extended with additional processes (i.e. including a second order term in the PDE) to more accurately describe the true settling behaviour [Plósz et al., 2007, Bürger et al., 2011]. However, the latter aspect requires the selection of a set of constitutive functions that are able to describe the dynamics of each of these processes. Although advanced 1-D models that also account for second-order processes become increasingly established, the application of these models for operation and control of WWTPs is hampered by their calibration. In order to obtain a 1-D model that can be used for operation and control, it is imperative to select adequate constitutive functions that are able to describe the dynamics of hindered and compression settling and to develop a calibration protocol for their parameters.

As hindered settling is active both in the hindered settling regime and in the compression regime (with compression working as a force against hindered settling), it follows that the hindered settling velocity should be the first function to examine. An inadequate choice for this function can after all impede the selection and calibration of the remaining functions. Literature on hindered settling is dominated by exponential and power law functions with the exponential functions being most established in commonly used layer models. However, since hindered settling is the only settling process considered in many of these layer models, the exponential relation may not have been selected for its superiority in describing the phenomenon of hindered settling as such but for its overall performance including the ability to partially compensate for missing phenomena. Independent analyses on the same data [De Clercq et al., 2008, Diehl, 2015] found that an exponential expression for the hindered settling velocity in combination with an expression for compression settling was not able to describe experimental batch settling data whereas a power-law function improved the model's ability to capture the data. Moving to advanced settler models which try to explicitly account for each phenomenon separately thus requires to re-evaluate available hindered settling functions.

Moreover, a thorough understanding of the different settling regimes requires more detailed data than standard 45 min batch settling curves. Hence, the current work evaluates the validity of different constitutive functions for hindered settling based on detailed data of long-term batch experiments.

2 Material and methods

2.1 1-D SST model

An advanced 1-D model which allows improved and more realistic simulation of secondary clarifiers has been presented by Bürger et al. [2011, 2012]. This model is called the Bürger-Diehl model. All implementation details can be found in Bürger et al. [2013]. The strength of the Bürger-Diehl SST model is twofold. It presents a numerical scheme that is consistent with current PDE-theory to ensure that numerical solutions approximate the PDE solutions and it allows the modeller to account for several phenomena (such as hindered settling, sludge compression and inlet dispersion) in a modular way making it very flexible in its application. Eq. 1 indicates the different constitutive functions that need to be defined (v_{hs} - hindered settling, d_{comp} - compression settling, d_{disp} - inlet dispersion). Each of these functions can easily be updated or replaced whenever research provides further insight in any of these phenomena. As gravitational settling and compression settling are the governing processes to predict important operating variables such as SBH and recycle concentration, the selection of a constitutive function for d_{disp} will not be considered here. Note that the compression function in Eq. 1 is written in its most general form as all analyses and conclusions in this work are made independently of any specific model structure for d_{comp}.

$$
\begin{aligned}
\frac{\partial X}{\partial t} = & \\
& - \frac{\partial}{\partial z}\left(v_{\mathrm{c}}(z,t)\, X\right) && \text{convective flow} \\
& - \frac{\partial}{\partial z}\left(v_{\mathrm{hs}}(X)\, X\right) && \text{hindered settling} \\
& + \frac{\partial}{\partial z}\left(d_{\mathrm{comp}}(X)\frac{\partial X}{\partial z}\right) && \text{compression settling} \\
& + \frac{\partial}{\partial z}\left(d_{\mathrm{disp}}(z, Q_{\mathrm{f}}(t))\frac{\partial X}{\partial z}\right) && \text{inlet dispersion} \\
& + \frac{Q_{\mathrm{f}}(t) X_{\mathrm{f}}(t)}{A}\delta(z) && \text{incoming feed flow}
\end{aligned}
\tag{1}
$$

2.2 Constitutive functions for hindered settling

Four hindered settling functions are considered in the current analysis: the most commonly used exponential functions by Vesilind [1968] (Eq. 2) and Takács et al. [1991] (Eq. 3), the power-law function by Cole [1968] (Eq. 4) which gave satisfactory results in the work by De Clercq et al. [2008] and finally a power-law function proposed by Diehl [2015] (Eq. 5). The latter selected this power-law function based on an extensive study to identify an expression for v_{hs} by solving an inverse problem. In these equations X represents the sludge concentration and V_0, r_{V}, r_{H}, r_{P}, k, n, $\bar{X}$ and q are positive parameters to be calibrated.

$$v_{\mathrm{hs}}(X) = V_0\, \mathrm{e}^{-r_{\mathrm{V}} X} \tag{2}$$

$$v_{\mathrm{hs}}(X) = V_0\, \left(\mathrm{e}^{-r_{\mathrm{H}} X} - \mathrm{e}^{-r_{\mathrm{P}} X}\right) \tag{3}$$

$$v_{\mathrm{hs}}(X) = k\, X^{-n} \tag{4}$$

$$v_{\mathrm{hs}}(X) = \frac{V_0}{1 + \left(\frac{X}{\bar{X}}\right)^q} \tag{5}$$

2.3 Experimental data

The analysis of these constitutive functions is based on two sets of batch settling data. The first data set was collected by De Clercq et al. [2005] who performed in-depth batch experiments by means of a radio-tracer. These experiments were carried out with sludge from the WWTP of Destelbergen (Belgium) at three different initial concentrations (2.40, 3.23 and 4.30 g/l) resulting in three sets of concentration profiles during 6 hours of settling (referred to as Data set 1).

The second set of data was collected by Locatelli et al. [2015] who applied an ultrasonic velocity profile technique to measure settling velocities within the sludge blanket without disturbing it. The measurements were carried out by an ultrasonic transducer which was installed above a batch settling column to conduct vertical measurements of the settling velocity. The ultrasonic velocity profile technique was originally developed by Takeda [1995] who reported the following specifications: a spatial resolution of 0.75 mm, a spatial measurement error of 1.1%, a velocity resolution of 0.75 mm/s and a velocity measurement error of 3.5%, indicating that this methodology provides accurate measurements of the settling velocity throughout the sludge blanket. Locatelli et al. [2015] performed experiments with sludge from the WWTP of Rosheim (France) at 6 different initial concentrations (1.5, 2.7, 3.2, 4.0, 4.6 and 5.6 g/l). As the initial concentration for each experiment is known and the settling velocities are measured, the settling fluxes can be calculated at each time from which changes in concentration can be tracked in order to determine the concentration profile. The experiments thus resulted in 6 sets of velocity and concentration profiles during approximately 1 hour of settling (referred to as Data set 2). Moreover, this data set was extended with one additional experiment at a concentration of 3.9 g/l where the settling process was monitored during 22 hours of settling.

2.4 Parameter estimation

The optimal parameters for the constitutive functions in the Eqn 2-5 are determined through the minimisation of an objective function that quantifies the quality of the model's fit to the experimental data. For the calibration exercise in this work the Sum of Squared Errors (SSE) calculated by Eq. 6 is selected as objective function. In this equation N is the number of data points, y_i the measured value at time i and $\hat{y}_i(\theta)$ the corresponding model prediction for a certain parameter set θ.

$$\mathrm{SSE} = \sum_{i=1}^{N} (y_i - \hat{y}_i(\theta))^2 \tag{6}$$

Finding the optimal parameter set that corresponds to a minimum value for the objective function requires an efficient search of the parameter space. As a highly accurate solution to the optimisation problem is not required (since the data are already subjected to measurement noise), the simple and robust Simplex iteration algorithm was used here [Nelder and Mead, 1965].

2.5 Confidence intervals

To check the quality of the parameter estimations, confidence intervals are calculated. The confidence intervals can be determined by using the error covariance matrix $C_H(\hat{\theta})$. However, calculating this matrix requires the inverse of the Hessian of the objective function which can be quite cumbersome to compute. Therefore, the inverse of the Fisher Information Matrix (FIM) is used as an approximation of the error covariance matrix [Asprey and Macchietto, 2000]. The FIM can easily be calculated through Eq. 7 as it only requires information on the measurement error covariance matrix and the local sensitivity functions.

$$\mathrm{FIM} = \sum_{i=1}^{N} \left(\frac{\partial y}{\partial \theta}\right)_i' Q_i \left(\frac{\partial y}{\partial \theta}\right)_i \tag{7}$$

In this equation $\left(\frac{\partial y}{\partial \theta}\right)_i$ represents an $n_\mathrm{m} \times n_\mathrm{p}$ matrix containing the local sensitivity functions for the m model outputs to the p model parameters at the time corresponding to measurement i. The measurement error covariance matrix Q_i (Eq. 8) is a diagonal matrix containing information on the measurement errors (σ_m) for each variable at the measurement times.

$$Q_\mathrm{i} = \mathrm{diag}\left(\frac{1}{\sigma_1^2(i)}, \ldots, \frac{1}{\sigma_m^2(i)}\right) \tag{8}$$

The local sensitivity functions are computed by a numerical approximation as described in Eq. 9. In this expression y is a model output and θ a model parameter. We may calculate $\Delta\theta$ by $\xi\,\theta$, where ξ is the pertubation factor. Here, a pertubation factor of 10^{-5} is used to ensure an adequate approximation of non-linearities in the model without risking numerical errors.

$$\frac{\partial y(t;\theta)}{\partial \theta} \approx \frac{y(t;\theta+\Delta\theta) - y(t;\theta)}{\Delta\theta} \tag{9}$$

Finally, the confidence intervals of the estimated parameters can be calculated from the FIM with Eq. 10, where t^{α}_{N-p} is the two-tailed t-distribution, with N the number of data points, p the number of estimated parameters and α the significance level and where $\mathrm{FIM}^{-1}(j,j)$ is the element on row j and column j of the inverse of the FIM.

$$\delta_i = \pm t^{\alpha}_{N-p}\sqrt{\mathrm{FIM}^{-1}(j,j)} \tag{10}$$

2.6 Akaike's Information Criteria (AIC) for model selection

The performance of the selected constitutive functions with respect to the experimental data can be quantified with a criterion for model selection such as AIC [Burnham and Anderson, 2004]. This criterion assesses different model structures taking into consideration their goodness of fit as well as their complexity (Eq. 11). Lower values for AIC indicate a better model.

$$\mathrm{AIC} = N\log\left(\frac{\mathrm{SSE}}{N}\right) + 2p \tag{11}$$

In this expression N represents the number of data points, p the number of estimated model parameters and SSE the sum of squared errors between the measured and simulated values.

3 Results and discussion

3.1 Calibration of hindered settling functions

For each data set the settling parameters of Eqs. 2-5 are calibrated based on the slope of the linear descent of the sludge blankets (Figure 1) as this presents a measurement of the hindered settling velocity. Note that if the initial concentration is high enough for compression to occur at the very beginning of the experiment, the batch curve will no longer show a clear linear descent but will start to become convex. Hence, such data should not be used to estimate the hindered settling parameters.

This results in three data points for the first data set and 6 data points for the second data set. Subsequently, the parameters for the hindered settling functions are calibrated by minimising the SSE (Eq. 6) between the slopes from the experimental batch curves

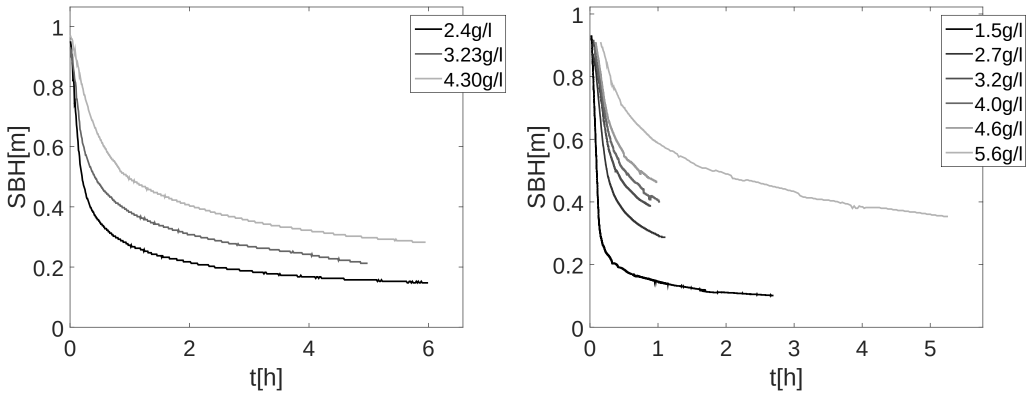

Figure 1: Measured SBH data from De Clercq et al. [2005] (left) and Locatelli et al. [2015] (right).

and (Eqn 2-5). The calibration of the hindered settling parameters is thus performed independently from the 1-D model. The resulting parameter sets and model fits are shown in Table 1 and Figure 2. In Data set 1 no confidence intervals could be calculated for the functions of Takács et al. [1991] and Diehl [2015] as for these functions three parameters need to be estimated based on only three data points. Hence, the t-test statistic in Eq. 10 cannot be calculated. The limited number of data points in Data set 1 also resulted in relatively large confidence intervals for the functions of Vesilind [1968] and Cole [1968]. Infinite confidence intervals are found with Data set 2 for the function of Takács et al. [1991] as the local sensitivity at the measured concentrations is zero for parameter r_P. This parameter is introduced to capture the decrease in settling velocity at low sludge concentrations (<1 g/l) which is observed in practice. However, introducing the additional parameter r_P in the hindered settling function is an artificial way to mimic the settling behaviour at low concentrations. In reality, at lower concentrations the sludge follows a completely different settling regime: discrete or flocculent settling where the sludge will no longer settle as a zone and no distinct sludge/water interface is formed making it impossible to record a batch settling curve. Hence, as no data are available in the operating range of the parameter r_P, this parameter cannot be properly estimated based on the available data resulting in the large confidence intervals from Table 1. A detailed discussion on modelling discrete settling behaviour is, however, outside the scope of this work.

3.2 Model selection

From Figure 2 it can be observed that all four hindered settling functions provide a good fit to the data. The performance of each hindered settling function is quantified with the AIC criterium in Table 2 (lowest AIC value for each data set indicated in bold). The power-law functions seem to perform better for the data of Locatelli et al. [2015] whereas the exponential functions perform better for the data of De Clercq et al. [2008]. In general all four functions produce AIC values in a similar range. Hence, it is difficult to select an optimal constitutive function based on these results.

However, a similar fit to the hindered settling data does not imply similar simulation results when implementing these functions in a 1-D model. Both at low and high concentration

Table 1: Optimal parameter values for the different hindered settling functions.

parameter	Data set 1	Data set 2
Vesilind		
V_0 [m/d]	256 ±81	310 ±11
r_V [l/g]	0.55 ±0.09	0.57 ±0.01
Takács		
V_0 [m/d]	4.38e5 ± NaN	310 ±∞
r_H [l/g]	0.872 ± NaN	0.573 ±∞
r_P [l/g]	0.873 ± NaN	0.025 ±∞
Cole		
k [kg/m^{2}h]	12.72 ±4.52	10.11 ±0.34
n [-]	1.701 ±0.294	1.423 ±0.027
Diehl		
V_0 [m/d]	277 ± NaN	440 ±165
$\bar{X}$ [g/m^3]	1411 ± NaN	931 ±0.301
q [-]	2.0831 ± NaN	1.73 ±0.112

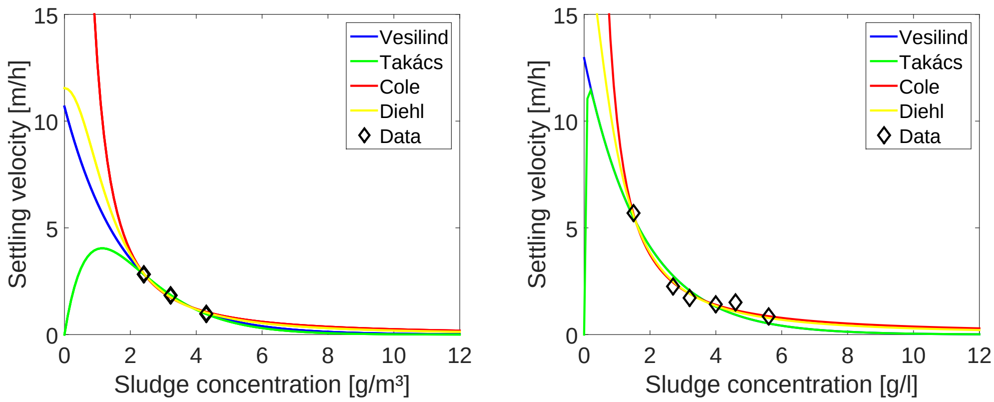

Figure 2: Comparison of different hindered settling functions calibrated to measured data of De Clercq et al. [2005] (left) and data of Locatelli et al. [2015] (right).

Table 2: Model selection of the different constitutive functions based on Akaike's Information Criterium.

	Vesilind	Takács	Diehl	Cole
Data set 1	-18.19	**-22.55**	-10.46	-11.04
Data set 2	-7.49	-5.49	-11.70	**-16.06**

ranges (where hindered settling does not occur or cannot be measured independently) differences between the presented functions can influence the predictions of the 1-D model. At low concentrations (roughly <1 g/l) discrete settling prevails (i.e. settling of individual flocs with size as the dominant influence rather than concentration). Including discrete settling in a 1-D model remains challenging and is outside of the scope of this work. At concentrations above approximately 6 g/l the differences in predicted settling velocities may seem small, however, from a relative perspective, they are profound. This is illustrated by the more detailed view of the predicted hindered settling velocities at high sludge concentrations in Figure 3. In reality, at these higher concentrations hindered settling will be slowed down by the formation of a network of particles that undergo a compressive force. The choice of the hindered settling function in this region will thus be important for the subsequent selection and calibration of a constitutive function for compression. This clearly illustrates the need for a critical analysis of the hindered settling velocity prior to the calibration of an advanced 1-D model including compression settling.

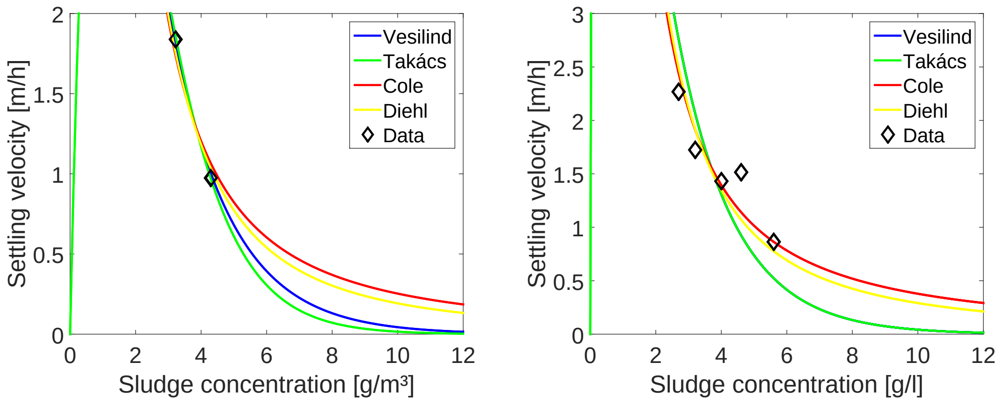

Figure 3: Detailed view of the predicted hindered settling functions calibrated to measured data of De Clercq et al. [2005] (left) and data of Locatelli et al. [2015] (right) to illustrate differences at high concentrations. Note that on the right graph, the functions of Vesilind and Takács overlap.

To assess the influence of the choice in hindered settling function on the 1-D model's behaviour in the compression region, the hindered settling functions are implemented in the 1-D model to see how they perform in predicting SBH data and velocity profiles. One exponential and one power-law function are selected for this further analysis. For the exponential functions, the function of Vesilind is selected as it shows almost identical behaviour to the function of Takács in the hindered and compression regions and requires only 2 parameters to be estimated. The power-law functions are represented by the function of Diehl. The latter choice was made from an implementation perspective. As can be seen from Figure 2 the function of Cole tends to infinity when X approaches 0. Hence, it has to be completed with another expression for low concentrations in order to be used in a 1-D model. This leads to additional parameters to calibrate.

3.3 Impact of different hindered settling functions on long-term SBH predictions

The hindered settling functions of Vesilind (Eq. 2) and Diehl (Eq. 5) are implemented in the Bürger-Diehl settler model (discretised with 90 layers) and used to simulate the experimental batch settling data. For the first data set, the resulting predictions of SBH are provided in Figure 4. (Note that no compression function is included for these simulations

($d_{\text{comp}} = 0$) as the goal of this work is to compare the behaviour of different hindered settling functions.)

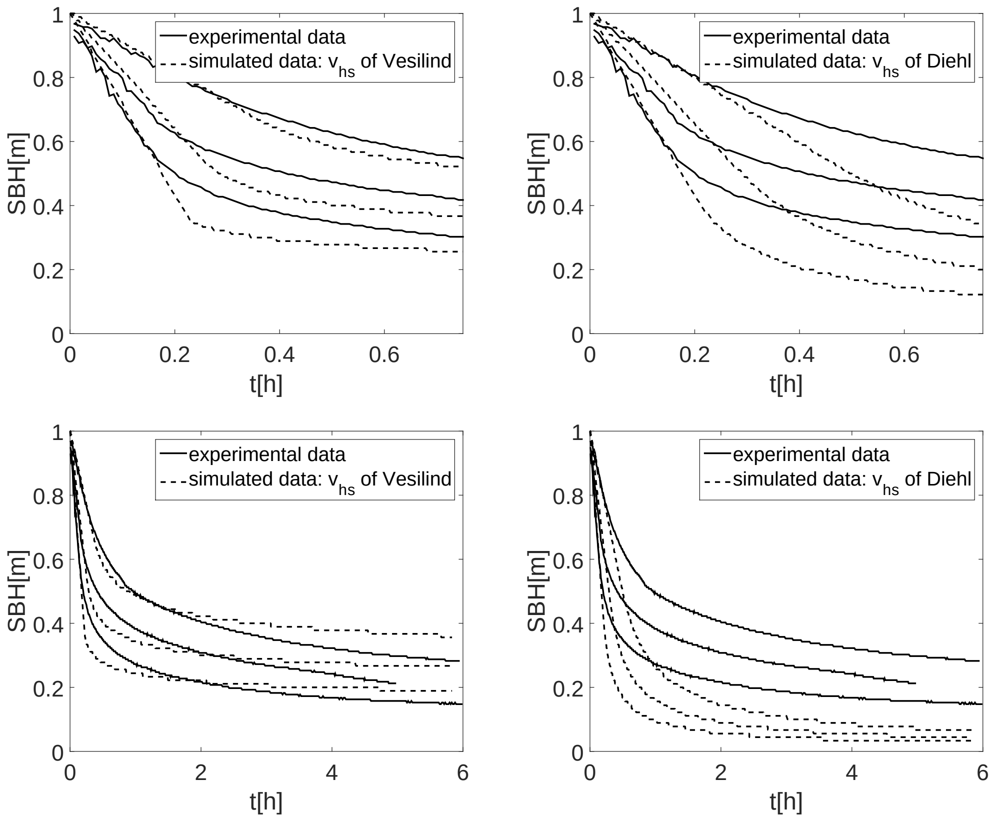

Figure 4: Measured SBHs (solid line - from De Clercq et al. [2005]) and predicted SBHs (dashed lines) by the Bürger-Diehl model with only hindered settling. Left: hindered settling function of Vesilind [1968], right: hindered settling function of Diehl [2015]. Results during first 45 min of batch settling (top) and 6 hours of settling (bottom).

First, the SBH predictions during the first 45 minutes of settling are investigated (shown at the top of Figure 4) as this is the typical duration of batch settling experiments. For both hindered settling functions the 1-D model performs well in predicting the initial linear descent of the sludge blanket but the models underpredict the SBH once the curve of the SBH starts to bend (corresponding to the onset of compression settling). From a modelling point of view, the underprediction is as expected. A 1-D model that only accounts for hindered settling will evidently overpredict the settling velocity at high concentrations and thus underpredict the SBH since it disregards the gradual compressibility that the formed particle network undergoes. To include more physical realism a compression function should be added which further slows down the settling velocity at high solids concentrations causing the sludge blanket to descend less rapidly.

From a more theoretical perspective, the behaviour of the different hindered settling functions can be explained as follows. Eqs. 2-5 all represent open functional forms and yield positive values of v_{hs} for all values of X (as long as $r_{\text{P}} < r_{\text{H}}$). As a consequence, when one solves a 1-D model without compressibility, the long-time result for each hindered settling

function will be that the SBH tends to zero while the bottom concentration tends to infinity (so called infinite thickening). By adding a compression function a proper balance between hindered settling and sludge compression is established which allows both SBH and bottom concentration to converge to a (more physically realistic) finite steady-state. However, the thickening dynamics for each of the functional forms in Eqs. 2-5 are considerably different. For example, in Figure 4 the underprediction is much larger for the hindered settling function of Diehl which can be expected as exponential functions are known to decay more rapidly than a rational function. These differences will have a significant influence on the consequent modelling of compression behaviour.
As the data set collected by De Clercq et al. [2005] provides data of up to 6 hours of settling the long term effects of different hindered settling functions and the consequences for subsequent compression modelling can be investigated in more detail. Therefore, the full measurements and simulation results for 6 hours of settling are shown at the bottom of Figure 4. These results clearly show that the 1-D model simulations with the exponential function of Vesilind perform much better in describing the overall trend in the experimental data. However, from a physical perspective these predictions do not make any sense. The predictions switch from an underprediction of the SBH (at t<2h) to an overprediction of the SBH (at t>2h) whereas we expect the SBH to be underpredicted over the entire experiment since no compression settling is taken into account. Hence, using an exponentially decaying function to describe hindered settling does not only account for hindered settling but already (unintentionally) compensates for the missing compressive behaviour (by predicting very low values for v_{hs} at high sludge concentrations - see Figure 3). Although this results in a model that performs relatively well in predicting the general trends, it will not succeed in capturing the true compression dynamics as compression is known to depend on the concentration gradient and its effect cannot be modelled by the convective flux [Concha and Bustos, 1987]. Moreover, the specific behaviour of the exponential function will even hamper further efforts to include more realism in 1-D models by accounting for other phenomena such as compression. Adding any type of compression function will inevitably further decrease the settling velocity at higher concentrations and thus increase the predicted SBHs. This is illustrated in Figure 5 (left) where a small amount of compression is added to the model (Eq. 12 with $\alpha = 0.1$ m^2/s^2, $X_{\text{crit}} = 6000$ g/m^3, $\rho_{\text{s}} = 1050$ kg/m^3 and $\rho_{\text{f}} = 998$ kg/m^3) and a clear increase in SBH can be seen for every time-interval where $X > X_{\text{crit}}$.

$$d_{\text{comp}}(X) = \begin{cases} 0 & \text{if } 0 \leqslant X < X_{\text{crit}} \\ \dfrac{\rho_{\text{s}} \cdot \alpha \cdot v_{\text{hs}}(X)}{g \cdot (\rho_{\text{s}} - \rho_{\text{f}})} & \text{if } X \geqslant X_{\text{crit}} \end{cases} \tag{12}$$

Hence, adding compression to the simulations with the exponential hindered settling function (Figure 4 - left) will move up the sludge blanket resulting in somewhat better predictions for t<2h but much worse predictions at t>2h.
The power-law function suggested by Diehl [2015] does perform as expected from a hindered settling function, i.e. an underprediction of the SBH over the entire time interval (Figure 4 - right). Adding a compression function will reduce this underprediction by increasing the SBH (Figure 5 - right). A power-law hindered settling function can thus be combined with a constitutive function for compression settling to obtain a more advanced 1-D model that accounts for both hindered and compression settling. The seemingly small differences in

v_{hs} predictions at high concentrations that were observed in Figure 2 cause considerable differences in SBH predictions and should not be ignored. Note that Figure 5 serves merely as an example of the general effect of extending a 1-D model with a compression function. Further prediction improvements require the selection and calibration of the compression function which have their proper challenges [De Clercq et al., 2008, Ramin et al., 2014] that are outside the scope of this contribution.

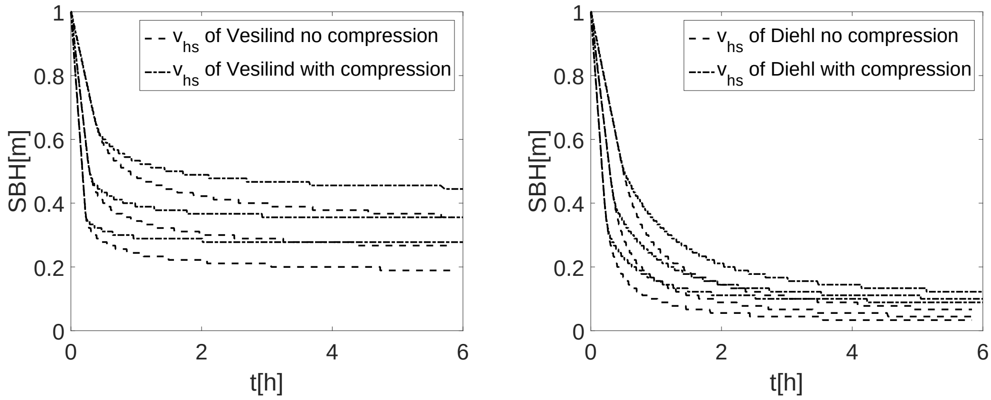

Figure 5: Simulation results illustrating the effect of extending the Bürger-Diehl model with a compression function. Predicted SBHs during 6 h of settling without compression (solid lines) and with compression (α = 0.1 m^2/s^2 and C_{crit} = 6000 g/m^3 - dashed lines). Left: hindered settling function of Vesilind [1968], right: hindered settling function of Diehl [2015].

3.4 Impact of different hindered settling functions on velocity profile predictions

The results above indicated that an exponential hindered settling function is not suitable to use in advanced 1-D SST models that aim to model the settling behaviour in more detail through the inclusion of compressive settling. These findings were further validated on the second data set by Locatelli et al. [2015] who provided experimental data of the settling velocity as well as the concentrations over the entire depth of the settling column during 22 hours of settling at an initial concentration of 3.9 g/l. The measured velocity and concentration profiles were compared to 1-D simulation results with the hindered settling functions of Vesilind [1968] and Diehl [2015] in the Bürger-Diehl model (discretised with 90 layers - $d_{\text{comp}} = 0$). Both experimental measurements and simulation results consist of data in three dimensions (velocity, time and depth). To facilitate the visualisation, the experimental data and the simulation results are presented in two dimensions in Figure 6. The left side of Figure 6 shows the evolution of the velocity inside the sludge blanket as a function of time, the right side of Figure 6 shows the evolution of the sludge blanket height as a function of time. The most important features of the batch profiles at the end of the experiment are further summarized in Table 3.
First, the simulation results with the commonly used exponential function of Vesilind are evaluated with respect to the experimental data. Comparing the SBHs between the experimental data and the model predictions leads to similar observations as in the previous

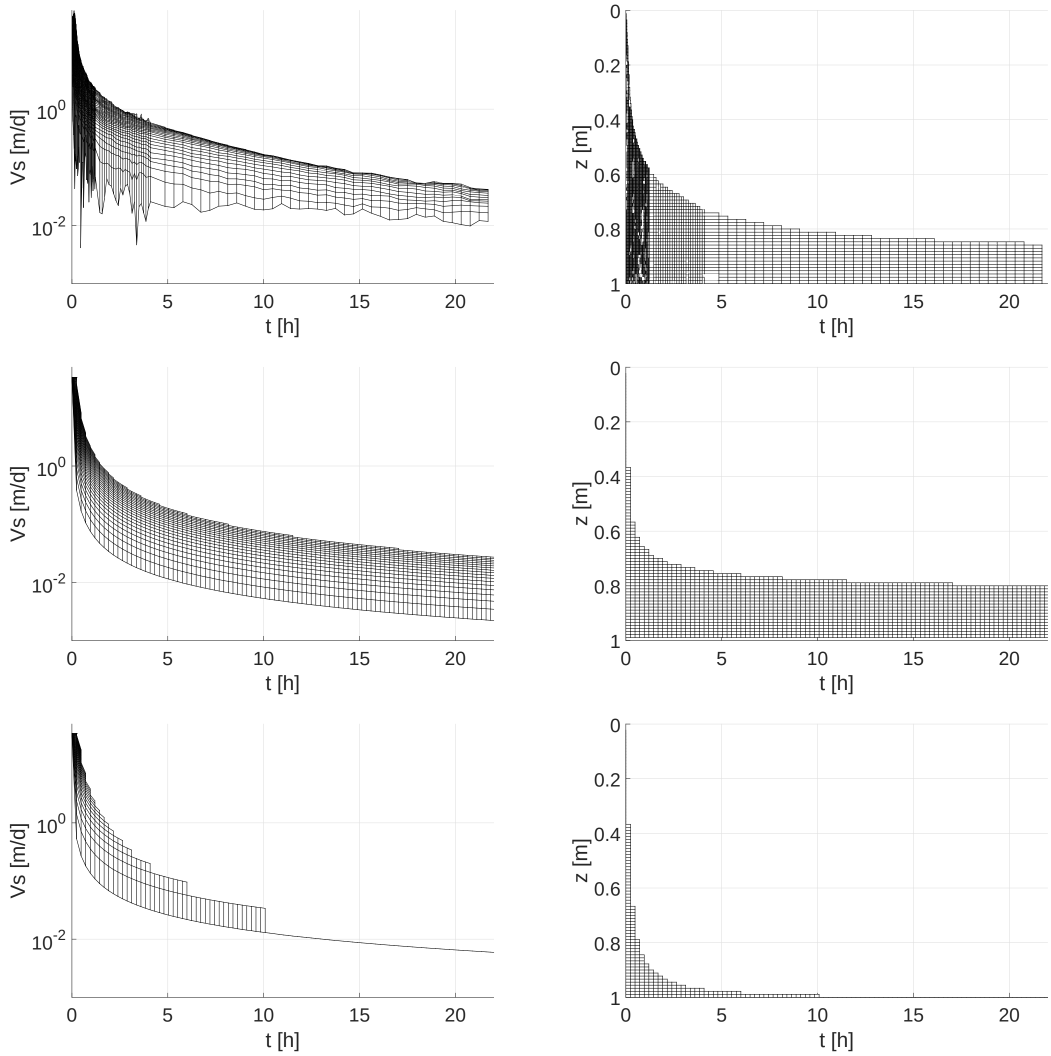

Figure 6: Left: velocity profiles inside the sludge blanket for a batch settling experiment at an initial concentration of 3.9 g/l (with the top curve representing the velocity at the top of the sludge blanket and the bottom curve the velocity of particles at the bottom of the settling column). Right: evolution of the SBH for a batch settling experiment at an initial concentration of 3.9 g/l. Experimental data (top), simulation with the settling velocity of Vesilind (center) and simulation results with the settling velocity of Diehl (bottom).

section. The model with the hindered settling velocity of Vesilind overpredicts the SBH at long settling times (predicted value of 0.21 m vs. a measured value of 0.15 m) indicating that it predicts settling velocities that are too low at higher sludge concentrations. This is further confirmed through the analysis of the velocities and concentration values. Where the 1-D model with the hindered settling velocity of Vesilind predicts a sludge concentration similar to the measured value at the height of the sludge blanket (10.5 g/l vs. 9.0 g/l), the predicted velocity is approx. 10 times smaller (0.0027 m/s vs. 0.02 m/s). At the bottom of the batch reservoir, this effect is even more pronounced with a measured concentration much higher than the predicted concentration (32 g/l vs. 21.8 g/l) even though the mea-

Table 3: Comparison of important features between the experimental data of Locatelli et al. [2015] and 1-D simulations with the calibrated hindered settling functions of Vesilind and Diehl after 22 hours of settling

	Exp. data	**1-D simulations**	
		v_{hs} Vesilind	v_{hs} Diehl
SBH [m]	0.15	0.21	0.02
$v_{s,bottom}$ [m/s]	0.0124	0.0022	0.0059
$v_{s,SBH}$ [m/s]	0.0380	0.0027	0.0200
X_{bottom} [g/l]	32.0	21.8	315.6
X_{SBH} [g/l]	9.0	10.5	31.6

sured velocity still exceeds the predicted value (0.0124 m/s vs. 0.0022 m/s). In summary, the prediction of low velocities at high concentrations in the commonly used exponential hindered settling functions hamper the thickening behaviour resulting in underpredictions of the bottom concentration and overpredictions of the SBH. This behaviour does not correspond with only hindered settling but indicates that some level of compression is accounted for in the exponential hindered settling functions. Consequently, these functions are unsuitable to use in advanced 1-D models as these models aim to include more realism by modelling the compression behaviour with an additional independent function and the known necessity to involve the gradient of the concentration.

The simulation results with the power-law hindered settling velocity of Diehl [2015] show a remarkably different behaviour. Here, the model predicts a sludge blanket of only two layers thick and an extremely concentrated bottom layer. Intuitively, this may seem strange, however, from a theoretical point of view this behaviour corresponds to what is expected from a hindered settling function. Without the presence of a compressive stress the particles can thicken at will resulting in an integral accumulation of particles in the bottom layer. Hence, if the modelling study requires a very simplified settler model (i.e. only accounting for hindered settling), an exponential settling function is clearly the best choice as it partially lumps other phenomena thus enforcing more realism. However, if these simplified settler models do not suffice for the objective of the modelling study (as is often the case for example in dynamic wet weather simulations in waste water treatment plant modelling) these models cannot simply be extended with additional phenomena such as compressive settling. A more advanced settler model that explicitly accounts for each phenomenon should be applied. For the latter models a power-law type function is found more appropriate in describing the hindered settling behaviour. This can subsequently be extended with a compression function for more accurate predictions of the sludge settling behaviour.

4 Conclusions

Two types of hindered settling functions were examined. The exponential relations as presented by Vesilind [1968] and Takács et al. [1991] and power-law type functions as presented by Cole [1968] and Diehl [2015]. The behaviour of these functions was evaluated based on long-term data of SBHs, concentration profiles and velocity profiles.

- The most commonly used exponential hindered settling functions produce contradictory results when confronted to long term batch data. At long settling times these exponential function underestimate the thickening behaviour of sludge resulting in SBH predictions that are too high and bottom concentrations that are too low. This behaviour does not correspond to the concept of hindered settling indicating that the exponential functions do not only account for hindered settling but partially compensate for compression in their model structure as well. By including compression in this way, further tuning is hampered as sludge compressibility cannot be disconnected from hindered settling nor is a hindered settling function able to describe the known dependency of sediment compressibility on the gradient of concentration. As a consequence, 1-D models with an exponential hindered settling function will only be able to provide a coarse approximation of the general trend but will not be able to describe the true dynamics of compression settling (which have been shown to have an important impact on SBH predictions and underflow dynamics).

- The power-law functions (known to decay less rapidly than exponential functions) predict higher settling velocities at high sludge concentrations. This results in almost perfect thickening behaviour (represented by extremely high bottom concentrations and a very thin sludge blanket). Hence, these functions perform as expected from a hindered settling function with a clear overprediction of the settling velocity in regions where compression is occurring. Compression can thus be included in the model as an additional process with the appropriate dynamics.

- Whereas the exponential functions may be an adequate choice for simplified settler models that only include a hindered settling function, they are not suitable for advanced settler models that aim to explicitly account for several phenomena. By implicitly compensating for compressive behaviour the use of an exponential hindered settling function will hamper further model extensions with compression dynamics. A power-law type function is much more appropriate to describe the hindered settling behaviour in an advanced 1-D model as it does not overcompensate for missing compression in its model structure. Hence, it is recommended to use a power law function in the further quest to solve the compression part of the settling problem.

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