LA SERENA NUMÉRICA II

Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena
La Serena, Chile, Enero 14 - 16, 2015

PROGRAM and ABSTRACTS

Contents

1 Introduction

The **Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales** has been organized in sequential talks of **45 and 30 minutes** length (40 and 25 minutes of presentation, respectively, and 5 minutes for questions and comments). All the talks will be given at Salón Multiuso, CETECFI (3th floor), **Facultad de Ingeniería**, Universidad de la Serena.

In the following pages we describe the corresponding program. In case of a multi-authored contribution, the speaker is underlined.

The organizers acknowledge financial support by:

- CONICYT-Chile through Project Anillo ACT 1118 ANANUM,
- Departamento de Matemáticas, Universidad de La Serena,
- Centro de Modelamiento Matemático (CMM), Universidad de Chile, and
- Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

In addition, we express our recognition and gratitude to all speakers for making **La Serena Numérica II** possible.

Organizing Committee
Raimund Bürger
Gabriel N. Gatica
Ricardo Oyarzúa
Héctor Torres

La Serena, January 2015

2 Wednesday, January 14

8.30-9.15 REGISTRATION

9.15-9.30 WELCOME SPEECH

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10.15-10.45 Dante Kalise: *High-order semi-Lagrangian schemes for static Hamilton-Jacobi-Bellman equations.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Sebastian U. Angel, Nelson O. Moraga, Mauricio J. Godoy: *A Darcy-Brinkman-Forchheimer porous model for alloys convective-diffusion solidification in molds.*

11.45-12.15 Michael Feischl, Thomas Führer, Dirk Praetorius, Ernst P. Stephan: *Multilevel methods for the hypersingular integral equation on locally refined triangulations.*

12.15-12.45 Sebastiano Boscarino, Raimund Bürger, Pep Mulet, Giovanni Russo, Luis M. Villada: *Linearly implicit IMEX Runge-Kutta methods for a class of degenerate convection-diffusion problems.*

12.45-15.00 LUNCH

[*Chairman*: R. BÜRGER]

15.00-15.45 Boris Andreianov, Mostafa Bendahmane, Alfio Quarteroni, Ricardo Ruiz-Baier: *Modelling, analysis, and numerical approximation of cardiac electromechanical interactions.*

15.45-16.15 Giordano Tierra, Juan P. Pavissich, Robert Nerenberg, Zhiliang Xu, Mark S. Alber: *Mathematical modeling of bacterial communities: Mechanical behavior of biofilms.*

16.15-16.45 Benjamín Barán: *Motherboard heat dissipation design using the Parareal method in PETSc.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Jessika Camaño, Ricardo Oyarzúa, Giordano Tierra: *Analysis of an augmented mixed-FEM for the Navier-Stokes problem.*

17.45-18.15 Michael Karkulik, Jens Markus Melenk: *Local high-order regularization and applications to hp-methods.*

18.15-18.45 Eligio Colmenares, Gabriel N. Gatica, Ricardo Oyarzúa: *Analysis of an augmented mixed–primal formulation for the stationary Boussinesq problem.*

19.30 WELCOME COCKTAIL

3 Thursday, January 15

[*Chairman*: G. GATICA]

9.30-10.15 Norbert Heuer, Michael Karkulik: *DPG analysis: adjoint problems and test norms.*

10.15-10.45 Fernando Henríquez, Carlos Jerez-Hanckes, Fernando Altermatt: *Boundary integral formulation for the electrical response of biological cells to external electrical stimulations.*

10.45-11.15 COFFEE BREAK

11.15-11.45 Alexis Jawtuschenko, Ariel L. Lombardi: *Anisotropic estimates for* H(**curl**)- *and* H(**div**)- *conforming elements on prisms and applications.*

11.45-12.15 Fernando Morales: *A discussion on the transmission conditions for saturated fluid flow through porous media with fractal microstructure.*

12.15-12.45 Bernardo Cockburn, Weifeng Qiu, Manuel Solano: *A priori error analysis for HDG methods in curved domains using extensions from polyhedral subdomains.*

12.45-15.00 OFFICIAL PICTURE/LUNCH

[*Chairman*: M. SOLANO]

15.00-15.45 Johnny Guzmán, Manuel Sanchez-Uribe, Marcus Sarkis: *Higher-order finite element methods for elliptic problems with interface.*

15.45-16.15 Gundolf Haase, Manfred Liebmann, Aurel Neic, Gernot Plank: *Many-core parallelization of AMG.*

16.15-16.45 Fernando Betancourt, Raimund Bürger, Stefan Diehl, Camilo Mejías: *Flux identification and efficient numerical simulation of clarifier-thickener units.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Gabriel N. Gatica, Filánder A. Sequeira: *Analysis of the HDG method for the Stokes-Darcy coupling.*

17.45-18.15 Ernesto Cáceres, Gabriel N. Gatica: *A mixed virtual element method for the Stokes problem.*

18.15-18.45 Rommel Bustinza: *An a priori error analysis of the HDG method for linear Stokes problem using a pseudostress–velocity formulation.*

20.30 CONFERENCE DINNER

4 Friday, January 16

[*Chairman*: R. OYARZÚA]

9.30-10.15 Antonio Baeza, Pep Mulet, David Zorío: *High order boundary extrapolation techniques for finite difference WENO schemes on complex domains.*

10.15-10.45 Mario Álvarez, Gabriel N. Gatica, Ricardo Ruiz-Baier: *Mixed-primal finite element approximation of a steady sedimentation-consolidation system.*

10.45-11.15 COFFEE BREAK

11.15-11.45 María del Carmen Martí, Pep Mulet: *Some computational techniques to improve component-wise finite-difference WENO schemes.*

11.45-12.15 Nicole Spillane, Victoria Dolean, et. al.: *Achieving robustness in domain decomposition methods.*

12.15-12.45 Dilberto da S. Almeida Júnior, Mauricio Sepúlveda C.: *Uniform stabilisation for a finite difference of the 1-d Timoshenko system.*

12.45-15.00 LUNCH

[*Chairman*: P. MULET]

15.00-15.45 Dmitry Kolomenskiy, Jean–Christophe Nave, Kai Schneider: *Space-time adaptive multiresolution techniques with a gradient-augmented level set method for advection equations.*

15.45-16.15 Gabriel N. Gatica, Ivana Šebestová: *Reconstruction-based a posteriori error estimation for the coupled Stokes-Darcy problem.*

16.15-16.45 Sudarshan K. Kenettinkara, Praveen Chandrashekhar, Veerappa Gowda: *A finite volume method for a two-phase multicomponent polymer flooding.*

16.45-17.15 COFFEE BREAK

17.15-17.45 Raimund Bürger, Pep Mulet, Lihki Rubio: *High-resolution schemes with polynomial viscosity matrices for multi-species kinematic flow models.*

17.45-18.15 Raimund Bürger, Ricardo Ruiz-Baier, Kai Schneider, Héctor Torres: *Numerical methods for multidimensional sedimentation models.*

18.15 CLOSING WORDS

5 Abstracts

Uniform stabilisation for a finite difference of the 1-d Timoshenko system*

Dilberto da S. Almeida Júnior† Mauricio Sepúlveda C.‡

Abstract

We consider some numerical schemes for the Timoshenko system with dissipative boundary conditions allowing the exponential stability and observability. It is known that for semi-discrete schemes obtained with finite difference or the standard finite element method there is not necessarily a uniform bound and therefore these numerical schemes are generally not observable for the limit $h \to 0$. This lack of boundary observability also happens to dissipative system in our study when the velocities of wave propagations are the same. However, we prove that there exists a subspace of solutions generated by the low frequencies of the discrete system where the numerical solutions are uniformly observable. Here we consider only the case in finite-difference and our proof relies on the method of numerical multipliers.

Key words:Timoshenko system; semi-discretizations; observability.

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*This research was partially supported by CONICYT-Chile through BASAL project CMM, Universidad de Chile, project Anillo ACT1118 (ANANUM), Fondecyt project 1140676; and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción.

†Department of Mathematics - Federal University of Pará Augusto Corrêa Street, 01, 66075-110, Bel em - Pará - Brazil, e-mail: `dilberto@ufpa.br`

‡CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `msepulveda@ci2ma.udec.cl`

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Mixed-primal finite element approximation of a steady sedimentation-consolidation system*

Mario A. Álvarez[†] Gabriel N. Gatica[‡] Ricardo Ruiz-Baier[§]

Abstract

This work is devoted to the mathematical and numerical analysis of a coupled flow and transport system typically encountered in continuum-based models of sedimentation-consolidation processes. These problems are relevant to a variety of applications including fluidized beds, clot formation within the blood, solid-liquid separation in wastewater treatement, and many others. The model problem focuses on the steady-state regime of the phenomenon and so the governing equations consist in the Brinkman equations with variable viscosity, written in terms of Cauchy stresses and bulk velocity of the mixture; coupled with a nonlinear advection – nonlinear diffusion equation describing the transport of the volume fraction. The variational approach is based on an augmented mixed formulation for the Brinkman problem and a primal weak form for the transport equation. Solvability of the coupling is established using classical fixed-point arguments, and we introduce an augmented mixed-primal Galerkin scheme based on Raviart-Thomas approximations of order k for the stress and piecewise continuous polynomials of order $k+1$ for velocity and volume fraction. We prove existence and uniqueness of the discrete problem also based on a fixed-point strategy, and we rigorously derive optimal error estimates in the natural norms. A few numerical tests illustrate the accuracy of the augmented mixed-primal finite element method, and the properties of the model.

Key words: Brinkman equations, nonlinear transport problem, augmented mixed–primal formulation, fixed point theory, thermal convection, sedimentation-consolidation process, finite element methods, a priori error analysis.

Mathematics subject classifications (2000): 65N30, 65N12, 76R05, 76D07, 65N15.

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[†]Sección de Matemática, Sede Occidente, Universidad de Costa Rica, San Ramón de Alajuela, Costa Rica, email: mario.alvarez@ucr.ac.cr.

[‡]CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: ggatica@ci2ma.udec.cl.

[§]Institut des Sciences de la Terre, Faculté des Géosciences et de l'Environnement, Géopolis UNIL-Mouline, Université de Lausanne, CH-1015 Lausanne, Switzerland, email: ricardo.ruizbaier@unil.ch.

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Modelling, analysis, and numerical approximation of cardiac electromechanical interactions*

Boris Andreianov† Mostafa Bendahmane ‡
Alfio Quarteroni§ Ricardo Ruiz-Baier ¶

Abstract

We present an overview of the numerical simulation of the interaction between cardiac electrophysiology, sub-cellular activation mechanisms, and macroscopic tissue contraction; that together comprise the essential elements of the electromechanical function of the human heart. We discuss the development of accurate mathematical models tailored for the simulation of the cardiac excitation-contraction mechanisms, which are primarily based on nonlinear elasticity theory and phenomenological descriptions of the mechano-electrical feedback. Here the link between contraction and the biochemical reactions at microscales is described by an active strain decomposition model. Then we turn to the mathematical analysis of a simplified version of the model problem consisting in a reaction-diffusion system governing the dynamics of ionic quantities, intra and extra-cellular potentials, and the elastodynamics equations describing the motion of an incompressible material. Under the assumption of linearized elastic behavior and a truncation of the updated nonlinear diffusivities, we are able to prove existence of weak solutions to the underlying coupled system and uniqueness of regular solutions. The proof of existence is based on a combination of parabolic regularization, the Faedo-Galerkin method, and the monotonicity-compactness method of J.L. Lions. A finite element formulation is also introduced, for which we establish existence of discrete solutions and show convergence to a weak solution of the original problem. We close with some numerical examples illustrating the convergence of the method and some features of the model.

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†Laboratoire de Mathématiques CNRS UMR 6623, Département de Mathématiques, Université de Franche-Comté, 16 route de Gray, 25030 Besançon Cedex, France; and Laboratoire Jacques-Louis Lions CNRS UMR 7598, Université Pierre et Marie Curie, 75252 Paris, France, e-mail: boris.andreianov@univ-fcompte.fr

‡Institut de Mathématiques de Bordeaux UMR CNRS 5251, Université Victor Segalen Bordeaux 2, F-33076 Bordeaux Cedex, France, e-mail: mostafa.bendahmane@u-bordeaux2.fr

§Modeling and Scientific Computing CMCS-MATHICSE-SB, École Polytechnique Fédérale de Lausanne EPFL, CH-1015 Lausanne, Switzerland; and on leave from MOX– Modellistica e Calcolo Scientifico, Dipartimento di Matematica "F. Brioschi" , Politecnico di Milano, via Bonardi 9, 20133 Milano, Italy, e-mail: alfio.quarteroni@epfl.ch

¶Institut des Sciences de la Terre, Faculté des Géosciences et de l'Environnement, Géopolis UNIL-Mouline, Université de Lausanne, CH-1015 Lausanne, Switzerland, email: ricardo.ruizbaier@unil.ch.

Key words: Electromechanical coupling, bidomain equations, finite element methods, coupled multiphysics, active strain, weak compactness method

Mathematics subject classifications (1991): 74F99, 35K57, 92C10, 65M60

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A Darcy-Brinkman-Forchheimer porous model for alloys convective-diffusion solidification in molds*

Sebastian U. Angel† Nelson O. Moraga† Mauricio J. Godoy†

Abstract

Two-dimensional unsteady fluid mechanics and heat transfer in liquid-solid phase changes are traditionally described by a system of five strongly coupled non-linear PDEs in the classical mathematical models. The use of the Darcy-Brinkman-Forchheimer porous model is presented in this talk to describe the fluid flow in the mushy zone. Temperature variable porosity and permeability in the mushy zone are incorporated into the macroscopic model to include relevant physical information found at the micro-scale level. We consider the solidification of an aluminum-silica alloy inside a thick walled mold. Unsteady heat conduction in the graphite mold coupled to diffusion in the solidified alloy and to the transient fluid mechanics and heat transport by convection and diffusion in the liquid phase and in the mushy region is described. Phase-change for binary Al-1.7Si alloy occurs between 650 and 550°C. Initial temperature of the melted alloy is 860°C while the mold is initially a 300°C, with convective cooling to the external air at 25°C. Boundary conditions of the third class (Robin type) are imposed on three mold walls and one of the Neumann type for the adiabatic bottom. The solution is obtained by using the PSIMPLER algorithm and the Finite Volume Method. Numerical experiments are performed to assess the effect of the convection in solidification processes with Rayleigh numbers Ra = 10^2, 10^3, 10^4 and 10^5. Results for the history of streamlines and isotherms calculated by the DBF model are compared with the solution obtained with the classical non-porous model. The DBF model incorporates new physical information by the additional terms, contributes to stabilize the numerical solution and reduces the CPU time needed to solve the problem.

Key words: Porous model, alloy solidification, finite volume method, sequential algorithm.

Mathematics subject classifications (2010): 80A22, 35K20, 65M60, 65N30, 76S05.

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†Departamento de Ingeniería Mecánica, Universidad de La Serena, Chile, email: nmoraga@userena.cl

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High order boundary extrapolation techniques for finite difference WENO schemes on complex domains *

Antonio Baeza† Pep Mulet‡ David Zorío§

Abstract

Finite difference WENO schemes [7, 9] have become an efficient method for the approximate solution of multidimensional hyperbolic conservation laws. These schemes follow a method of lines strategy, for which the spatial discretization is obtained by numerical differentiation of reconstruction of fluxes. Higher order accuracy can then be obtained from highly accurate reconstructions and these can be quite readily designed as long as the underlying mesh is an equispaced Cartesian mesh. In this context, the application of suitable numerical boundary conditions for hyperbolic conservation laws on domains with complex geometry has become a problem with certain difficulty that has been tackled in different ways according to the nature of the numerical methods and mesh type([3, 10, 11, 12]. In this presentation, we propose an extrapolation technique on structured Cartesian meshes (which, as opposed to non-structured meshes, can not be adapted to the morphology of the domain boundary) of the information in the interior of the computational domain to ghost cells. This technique is based on the application of Lagrange interpolation with a previous filter for the detection of discontinuities that permits a data dependent extrapolation, with higher order at smooth regions and essentially non oscillatory properties near discontinuities. Some simulations with the Euler equations of gas dynamics on relatively complex domains ([13, 3]) are presented in order to assess the capabilities of the proposed techniques.

Key words: Hyperbolic conservation laws, finite difference WENO schemes, boundary extrapolation, Euler equations.

Mathematics subject classifications (1991): 65N06, 65N40, 65N50

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†Dpt. Matemàtica aplicada, Universitat de València, Av. Vicent Andrés Estellés s/n, 46007, Burjassot, València, Spain email: Antonio.Baeza@uv.es

‡Dpt. Matemàtica aplicada, Universitat de València, e-mail: mulet@uv.es

§Dpt. Matemàtica aplicada, Universitat de València, e-mail: David.Zorio@uv.es

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Motherboard heat dissipation design using the Parareal method in PETSc

Benjamín Barán *

Abstract

This work presents a parallel implementation of the Parareal method using PETSc (Portable, Extensible Toolkit for Scientific Computation) to solve a typical motherboard heat dissipation design problem that can be viewed as a parabolic partial differential equation with known boundary conditions and initial state, where the minimized cost function relates the controller energy usage and the approximation of the solution to an optimal known function. The equations that model the process are discretized with Finite Elements in space and Finite Differences in time. After discretization in space and time, the problem is transformed to a huge linear system of algebraic equations that is solved by the Conjugate Gradient method. The Parareal preconditioner is implemented to speed up the convergence of the Conjugate Gradient. The main advantage in using the Parareal method in this parallel implementation in PETSc is to speed up the resolution time, when comparing to implementations that only use the Conjugate Gradient or GMRES methods. The implementation developed in this work offers a parallelization relative efficiency for the strong scaling that is approximately 70% each time the process count doubles, while for the weak scaling it is 75% each time the process count doubles for a constant solution size per process and up to 96% each time the process count doubles for a constant data size per process.

Key words: motherboard design, parallel implementation in PETSc, Pararreal preconditioner, Finite Elements in space and Finite Difference in time, parallelization efficiency.

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*East National University (UNE) and National University of Asuncion (UNA) - Paraguay. Email: bbaran@pol.una.py

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LA SERENA NUMERICA II

Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Flux identification and efficient numerical simulation of clarifier-thickener units*

FERNANDO BETANCOURT† RAIMUND BÜRGER‡

STEFAN DIEHL § CAMILO MEJÍAS¶

Abstract

Mathematical models for the simulation of batch settling and continuous clarifier-thickeners can usually be expressed as a convection-diffusion partial differential equation (PDE). Reliable numerical methods require that the nonlinear flux function of this PDE has been identified for a given material. This contribution simulates a continuous process of thickening with a numerical model adapted from wasterwater treatment [2], that involves a discontinuous flux. Results are compared with experimental data from JRI labs, a consulting of copper mining specialised in thickener process and unit areas. This contribution is an extension of [1] and is based on the treatment of an inverse problem published by R. Bürger and S. Diehl in [3] for the flux identification in the case of a suspension that shows no sediment compressibility. Moreover, we present a simulation of the complete process based on experimental data obtained from a batch settling test. The experimental information determines the choice of the batch flux density function. We then proceed to solve the associated problem by Godunov's first-order method.

Key words: Continuous sedimentation, simulation model, Solid-liquid separation, Flux identification.

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†Departamento de Ingeniería Metalúrgica, Facultad de Ingeniería, Universidad de Concepción, Casilla 160-C, Concepción, Chile. E-mail: `fbetancourt@udec.cl`

‡CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `rburger@ing-mat.udec.cl`

§Centre for Mathematical Sciences, Lund University, P.O. Box 118, S-221 00 Lund, Sweden. E-Mail: `diehl@maths.lth.se`

¶CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `cmejias@ing-mat.udec.cl`

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Linearly implicit IMEX Runge-Kutta methods for a class of degenerate convection-diffusion problems*

Sebastiano Boscarino† Raimund Bürger‡ Pep Mulet§
Giovanni Russo¶ Luis M. Villada‖

Abstract

Multi-species kinematic flow models with strongly degenerate diffusive corrections give rise to systems of nonlinear convection-diffusion equations of arbitrary size. Applications of these systems include models of polydisperse sedimentation and multi-class traffic flow. Implicit-explicit (IMEX) Runge-Kutta (RK) methods [1] are suitable for the solution of these convection-diffusion problems since the stability restrictions, coming from the explicitly treated convective part, are much less severe than those that would be deduced from an explicit treatment of the diffusive term. These schemes usually combine an explicit Runge-Kutta scheme for the time integration of the convective part with a diagonally implicit one for the diffusive part. In [4] a scheme of this type is proposed, where the nonlinear and non-smooth systems of algebraic equations arising in the implicit treatment of the degenerate diffusive part are solved by smoothing of the diffusion coefficients combined with a Newton-Raphson method with line search. This nonlinearly implicit method is robust but associated with considerable effort of implementation and possibly CPU time. To overcome these shortcomings while keeping the advantageous stability properties of IMEX-RK methods, a second variant of these methods is proposed, in which the diffusion terms are discretized in a way that more carefully distinguishes between stiff and nonstiff dependence [3], such that in each time step only a linear system needs to be solved still maintaining high order accuracy in time, which makes these methods much simpler to implement. In a series of examples

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†Department of Mathematics and Computer Science, University of Catania, 95125 Catania, Italy. E-Mail: boscarino@dmi.unict.it

‡CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: rburger@ing-mat.udec.cl

§Departament de Matemàtica Aplicada, Universitat de València, Av. Dr. Moliner 50, E-46100 Burjassot, Spain. E-Mail: mulet@uv.es

¶Department of Mathematics and Computer Science, University of Catania, 95125 Catania, Italy. E-Mail: russo@dmi.unict.it

‖Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile. E-Mail: lvillada@ubiobio.cl

of polydisperse sedimentation [2] and multi-class traffic flow [5] it is demonstrated that these new linearly implicit IMEX-RK schemes approximate the same solutions as the nonlinearly implicit versions, and in many cases these schemes are more efficient.

Key words: Implicit-explicit Runge-Kutta schemes, degenerate convection-diffusion equations, linearly implicit methods, polydisperse sedimentation, multiclass traffic flow.

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Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Numerical methods for multidimensional sedimentation models *

Raimund Bürger† Ricardo Ruiz-Baier ‡
Kai Schneider§ Héctor Torres ¶

Abstract

We study multidimensional sedimentation models which are given by the Navier-Stokes equations, these equations describe the motion of the involved mixture coupled to a parabolic (hyperbolic) equation for the evolution of the local solid concentration. We are interested in the numerical simulation of the sedimentation of monodisperse suspensions. We propose numerical schemes [1, 2, 3] for the numerical solution of multidimensional models, describing the phenomenon in different configurations. Some numerical experiments illustrate properties of the model and the satisfactory performance of the proposed methods.

Key words: Finite volume element methods, sedimentation process, adaptive multiresolution scheme.

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†CI²MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: rburger@ing-mat.udec.cl

‡Institut des Sciences de la Terre, Faculté des Géosciences et de l'Environnement, Géopolis UNIL-Mouline, Université de Lausanne, CH-1015 Lausanne, Switzerland, email: ricardo.ruizbaier@unil.ch.

§M2P2CNRS, Aix-Marseille Université 39, rue Frédéric Joliot-Curie, 13453 Marseille Cedex 13, France, email: kschneid@cmi.univ-mrs.fr

¶Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, e-mail: htorres@userena.cl.

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

High-resolution schemes with polynomial viscosity matrices for multi-species kinematic flow models*

Raimund Bürger† Pep Mulet‡ Lihki Rubio§

Abstract

In this work, we present a class of fast first order finite volume solvers, called PVM (polynomial viscosity matrix), for conservative hyperbolic systems. They are defined in terms of viscosity matrices computed by a suitable polynomial evaluation of a Roe matrix. These methods have the advantage that they only need some information about the eigenvalues of the system to be defined, and no spectral decomposition of a Roe matrix is needed. As a consequence, they are faster than the Roe method. On the other hand, because we propose to use a first order finite volume solvers, we analyze the use of high-order reconstruction method WENO and MUSCL [3], each of these used for the reconstruction of ows and states respectively, for obtain as result a high-resolution schemes with polynomial viscosity matrices economic in the sense of PVM methods [4]. The numerical tests presented here, the performances of the numerical schemes and compared with each others is performed for multi-species kinematic ow models. These models have been studied for example in [1] and [2].

Key words: PVM, Multi-Species Kinematic Flow Models, WENO, MUSCL.

Mathematics subject classifications (1991): 665N30, 65N38, 76D07, 76M10, 76M15

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†CI^2MA and Departamento de Ingenierá Matemática, Universidad de Concepción, Casilla 160-C, Concepci ón, Chile, email: rburger@ing-mat.udec.cl

‡Department of Applied Mathematics, Faculty of Mathematics, University of Valencia, España, email: mulet@uv.es

§CI^2MA and Departamento de Ingenierá Matemática, Universidad de Concepción, Casilla 160-C, Concepci ón, Chile, email: lrubio@ci2ma.udec.cl

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

An a priori error analysis of the HDG method for linear Stokes problem using a pseudostress–velocity formulation*

Rommel Bustinza†

Abstract

In this talk we propose and analyze a hybridizable discontinuous Galerkin (HDG) formulation for the Stokes problem of incompressible fluid flow. The main unknowns here are the velocity, the so called pseudostress (cf. [1, 2, 6]), and the trace of velocity on the skeleton of the considered mesh of the domain. We remark that the referred trace unknown will be approximated by the well–known numerical flux of velocity, which allows us to rewrite the scheme as one defined on the skeleton of the mesh, reducing significatively the degrees of freedom. This can be done thanks to the linearity of the problem, which makes it possible the introduction of suitable local solvers. The corresponding projection–based a priori error analysis is developed following and/or adapting the ideas given in [3, 5], and ensures the expected (optimal) rates of convergence.

Key words: mixed-FEM, HDG, Stokes problem

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†CI^2MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `rbustinz@ing-mat.udec.cl`

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Departamento de Matemática, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

A mixed virtual element method for the Stokes problem*

Ernesto Cáceres† Gabriel N. Gatica‡

Abstract

In this paper we introduce and analyze a virtual element method (VEM) for a mixed variational formulation of the Stokes problem in which the pseudostress and the velocity are the only unknowns, whereas the pressure is computed via a postprocessing formula. We first recall the corresponding continuous variational formulation, and then, following the basic principles for mixed-VEM, define the virtual finite element subspaces to be employed, introduce the associated interpolation operators, and provide the respective approximation properties. In particular, the latter includes the estimation of the interpolation error for the pseudostress variable measured in the $\mathbb{H}(\mathbf{div})$-norm. Next, and in order to define calculable discrete bilinear forms, we propose a new local projector onto a suitable space of polynomials, which takes into account the main features of the continuous solution and allows the explicit integration of the terms involving the deviatoric tensors. The uniform boundedness of the resulting family of local projectors and its approximation properties are also established. In addition, we show that the global discrete bilinear forms satisfy all the hypotheses required by the Babuška-Brezzi theory. In this way, we conclude the well-posedness of the actual Galerkin scheme and derive the associated a priori error estimates for the virtual solution as well as for the fully computable projection of it. Finally, several numerical results illustrating the good performance of the method and confirming the theoretical rates of convergence are presented.

Key words: Stokes equations, virtual element method, a priori error analysis

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 76D07

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†CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `ecaceresv@udec.cl`

‡CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `ggatica@ci2ma.udec.cl`

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Analysis of an augmented mixed-FEM for the Navier-Stokes problem*

Jessika Camaño† Ricardo Oyarzúa‡ Giordano Tierra§

Abstract

In this work we propose and analyze a new augmented mixed finite element method for the Navier-Stokes problem. Our approach is based on the introduction of a "nonlinear-pseudostress" tensor linking the pseudostress tensor with the convective term, which leads to a mixed formulation with the nonlinear-pseudostress tensor and the velocity as the main unknowns of the system. Further variables of interest, such as the fluid pressure, the fluid vorticity and the fluid velocity gradient, can be easily approximated as a simple postprocess of the finite element solutions with the same rate of convergence. The resulting mixed formulation is augmented by introducing Galerkin least-squares type terms arising from the constitutive and equilibrium equations of the Navier-Stokes equations and from the Dirichlet boundary condition, which are multiplied by stabilization parameters that are chosen in such a way that the resulting continuous formulation becomes well-posed. Then, the classical Banach's fixed point Theorem and Lax-Milgram's Lemma are applied to prove well-posedness of the continuous problem. Similarly, we establish well-posedness and the corresponding Cea's estimate of the associated Galerkin scheme considering any conforming finite element subspace for each unknown. In particular, the associated Galerkin scheme can be defined by employing Raviart-Thomas elements of degree k for the nonlinear-pseudostress tensor, and continuous piecewise polynomial elements of degree $k+1$ for the velocity, which leads to an optimal convergent scheme. In addition, we provide two iterative methods to solve the corresponding nonlinear system of equations and analyze their convergence. Finally, several numerical results illustrating the good performance of the method are provided.

Key words: Navier-Stokes, mixed finite element method, augmented formulation, Raviart-Thomas elements

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†Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Casilla 297, Concepción, Chile, and CI²MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `jecamano@ucsc.cl`

‡Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile, and CI²MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: `royarzua@ubiobio.cl`

§Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Prague 8, 186 75, Czech Republic, e-mail: `gtierra@karlin.mff.cuni.cz`.

Mathematics subject classifications (1991): 65N15, 65N30, 76D05, 76M10

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

A priori error analysis for HDG methods in curved domains using extensions from polyhedral subdomains

BERNARDO COCKBURN * WEIFENG QIU† MANUEL SOLANO‡

Abstract

We present an a priori error analysis of a technique ([3], [4]) that allows us to numerically solve diffusion problems with Dirichlet boundary conditions defined on curved domains Ω by using finite element methods defined in polyhedral subdomains $\mathsf{D}_h \subseteq \Omega$. We prove that the order of convergence in the L^2-norm of the approximate flux and scalar unknowns is optimal as long as the distance between the boundary of the original domain Γ and that of the computational domain Γ_h is of order h. We also prove that the L^2-norm of a projection of the error of the scalar variable superconverges with a full additional order when the distance between Γ and Γ_h is of order $h^{5/4}$ but with only half an additional order when such a distance is of order h. In addition, we present numerical experiments validating the theoretical results and showing that even when the distance between Γ and Γ_h is of order h, the above-mentioned projection of the error of the scalar variable can still superconverge with a full additional order.

Key words: HDG, curved domains, immerse boundary method

Mathematics subject classifications (2010): 65N30, 65M60

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*School of Mathematics, University of Minnesota, Minneapolis MN 55455, USA email: cockburn@math.umn.edu

†Department of Mathematics, City University of Hong-Kong, Kowloon Tong, Hong Kong e-mail: weifeqiu@cityu.edu.hk

‡CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, Chile e-mail: msolano@ing-mat.udec.cl

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LA SERENA NUMERICA II
Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemática, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Analysis of an augmented mixed–primal formulation for the stationary Boussinesq problem*

Eligio Colmenares[†] Gabriel N. Gatica[‡] Ricardo Oyarzúa[§]

Abstract

In this paper we propose and analyze a new mixed variational formulation for the stationary Boussinesq problem. Our method, which employs a technique previously applied to the Navier-Stokes equations, is based first on the introduction of a modified pseudostress tensor depending nonlinearly on the velocity through the respective convective term. Next, the pressure is eliminated, and an augmented approach for the fluid flow, which incorporates Galerkin type terms arising from the constitutive and equilibrium equations, and from the Dirichlet boundary condition, is coupled with a primal-mixed scheme for the main equation modeling the temperature. In this way, the only unknowns of the resulting formulation are given by the aforementined nonlinear pseudostress, the velocity, the temperature, and the normal derivative of the latter on the boundary. An equivalent fixed-point setting is then introduced and the corresponding classical Banach Theorem, combined with the Lax-Milgram Theorem and the Babuška-Brezzi theory, are applied to prove the unique solvability of the continuous problem. In turn, the Brouwer and the Banach fixed point theorems are utilized to establish existence and uniqueness of solution, respectively, of the associated Galerkin scheme. In particular, Raviart-Thomas spaces of order k for the pseudostress, continuous piecewise polynomials of degree $\leq k+1$ for the velocity and the temperature, and piecewise polynomials of degree $\leq k$ for the boundary unknown become feasible choices. Finally, we derive optimal a priori error estimates, and provide several numerical results illustrating the good performance of the augmented mixed-primal finite element method and confirming the theoretical rates of convergence.

Key words: Boussinesq equations, augmented mixed–primal formulation, fixed point theory, finite element methods, a priori error analysis

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[†]CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `eligio@ci2ma.udec.cl`

[‡]CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `ggatica@ci2ma.udec.cl`

[§]GIMNAP-Departamento de Matemática, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile, and CI^{2}MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `royarzua@ubiobio.cl`.

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 35Q79, 80A20, 76R05, 76D07

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Polynomial-degree-robust a posteriori estimates in a unified setting*

Alexandre Ern† Martin Vohralík‡

Abstract

We present equilibrated flux a posteriori error estimates in a unified setting for conforming, nonconforming, discontinuous Galerkin, and mixed finite element discretizations of the two-dimensional Poisson problem. Relying on the equilibration by mixed finite element solution of patchwise Neumann problems, the estimates are guaranteed, locally computable, locally efficient, and robust with respect to polynomial degree. Maximal local overestimation is guaranteed as well. Numerical experiments suggest asymptotic exactness for the incomplete interior penalty discontinuous Galerkin scheme. Details can be found in [1].

Key words: a posteriori error estimate, equilibrated flux, unified framework, robustness, polynomial degree, conforming finite element method, nonconforming finite element method, discontinuous Galerkin method, mixed finite element method

Mathematics subject classifications (1991): 35J20, 65N15, 65N30, 76M10, 76S05

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†Université Paris-Est, CERMICS (ENPC), 77455 Marne-la-Vallée, France, email: ern@cermics.enpc.fr

‡INRIA Paris-Rocquencourt, B.P. 105, 78153 Le Chesnay, France, e-mail: martin.vohralik@inria.fr

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Multilevel methods for the hypersingular integral equation on locally refined triangulations*

Michael Feischl[†] Thomas Führer[‡]
Dirk Praetorius[§] Ernst P. Stephan[¶]

Abstract

We consider the hypersingular integral equation for the 2D and 3D Laplacian. It is well-known that the condition number of the Galerkin matrix grows with the number of elements as well as the global mesh-size quotient $h_{\max}/h_{\min}$ as the mesh is (locally) refined. Therefore, the developement of *optimal* preconditioners is a necessary and important task. Here, optimality is understood in the sense that the resulting condition numbers are independent of the number of elements and the mesh-size. In this talk, we present results from [1], where we consider a (local) multilevel diagonal preconditioner. The basic idea of this preconditioner is to consider only newly created nodes in $\mathcal{T}_{\ell+1}\backslash\mathcal{T}_\ell$ plus some of their immediate neighbours for preconditioning. For uniform refinement, it was proved in [3] that multilevel diagonal preconditioners are optimal. On locally refined triangulations such a result was unknown. Basically, the proof of optimality consists of providing a stable subspace decomposition for the fractional order Sobolev space $H^{1/2}$ in the frame of additive Schwarz methods. Our analysis relies on an appropriate variant of the Scott-Zhang projection [2] and hierarchical properties of the mesh-refinement employed. Numerical examples on closed and open boundaries underline our theoretical results.

Key words: adaptivity, multilevel methods, hypersingular integral equation

Mathematics subject classifications (2010): 65N30, 65F08, 65N38

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†Institute for Analysis and Scientific Computing, Vienna University of Technology, Vienna, Austria, e-mail: `michael.feischl@tuwien.ac.at`

‡Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Santiago, Chile.

§Institute for Analysis and Scientific Computing, Vienna University of Technology, Vienna, Austria, e-mail: `dirk.praetorius@tuwien.ac.at`

¶Institute for Applied Mathematics, Leibniz University Hannover, Hannover, Germany, e-mail: `stephan@ifam.uni-hannover.de`.

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Analysis of the HDG method for the Stokes-Darcy coupling*

Gabriel N. Gatica† Filánder A. Sequeira‡

Abstract

In this talk we introduce and analyze a hybridizable discontinuous Galerkin (HDG) method for numerically solving the coupling of fluid flow with porous media flow. Flows are governed by the Stokes and Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We consider a fully-mixed formulation in which the main unknowns are given by the stress, the vorticity, the velocity, and the trace of the velocity, all them in the fluid, together with the velocity, the pressure, and the trace of the pressure in the porous medium. In addition, we enrich the finite dimensional subspace for the stress, in order to obtain optimally convergent approximations for all unknowns, as well as a superconvergent approximation of the trace variables. To do that, similarly as in previous papers dealing with development of the *a priori* error estimates, we use the projection-based error analysis in order to simplify the corresponding study. Finally, we provide several numerical results illustrating the good performance of the proposed scheme and confirming the optimal order of convergence provided by the HDG approximation.

Key words: coupling, Stokes equations, Darcy equations, mixed finite element method, hybridized discontinuous Galerkin method

Mathematics subject classifications (2012): 65N15, 65N30, 35L65, 74F10, 74S05

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†CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `ggatica@ci2ma.udec.cl`

‡Escuela de Matemática, Universidad Nacional de Costa Rica, Heredia, Costa Rica, email: `filander.sequeira@una.cr`. Present address: CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `fsequeira@ci2ma.udec.cl`.

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Reconstruction-based a posteriori error estimation for the coupled Stokes-Darcy problem*

Gabriel N. Gatica† Ivana Šebestová‡

Abstract

We derive a posteriori error estimates for the mixed finite element approximations for the coupled Stokes–Darcy problem. The transmission conditions are given by the conservation of mass, the balance of normal forces, and the Beavers-Joseph-Saffman law. In particular, the normal continuity of the velocity field on the interface between fluid flow and porous media flow is imposed strongly in the space for velocity field. For the weak imposition of this condition, it has been shown that combination of any pair of stable finite element methods for the pure Stokes and pure Darcy problems yields a convergent scheme for the coupled problem in Gatica et al. (2011) [6]. For the strong incorporation of that condition, the result has been extended in Márquez et al. (2014) [8]. Our analysis is based on a conforming velocity reconstruction and locally conservative flux reconstruction. The velocity reconstruction is defined to satisfy the normal continuity condition across the interface. The flux reconstruction is defined via mixed finite element approximations of local Neumann or Neumann–Dirichlet problems following an approach in Hannukainen et al. (2012) [7]. The derived a posteriori error estimates are proven to be reliable.

Key words: a posteriori error estimate, velocity and flux reconstruction, coupled Stokes–Darcy flow

Mathematics subject classifications (1991): 65N15, 65N30, 76D07

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†CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, e-mail: ggatica@ci2ma.udec.cl

‡CI^{2}MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: isebestova@ci2ma.udec.cl

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Higher-order finite element methods for elliptic problems with interface

Johnny Guzmán* Manuel Sanchez-Uribe† Marcus Sarkis‡

Abstract

We present higher-order piecewise continuous finite element methods for solving a class of interface problems in two dimensions. The method is based on correction terms added to the right-hand side in the standard variational formulation of the problem. We prove optimal error estimates of the methods on general quasi-uniform and shape regular meshes in maximum norms. In addition, we apply the method to a Stokes interface problem, adding correction terms for the velocity and the pressure, obtaining optimal convergence results.

*Division of Applied Mathematics, Brown University, Providence, RI, USA email: johnny_guzman@brown.edu

†Division of Applied Mathematics, Brown University, Providence, RI, USA e-mail: manuel_sanchez-uribe@brown.edu

‡Departament of Mathematics, Worcester Polytechnic University, Worecester, MA, USA e-mail: msarkis@wpi.edu

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Many-core parallelization of AMG*

Gundolf Haase† Manfred Liebmann Aurel Neic‡ Gernot Plank

Abstract
We developed our algebraic multigrid solvers (AMG) with special focus on cardiac electromechanics with the goal of simulating one heart beat as fast as possible. The overall systems consist of the bidomain equations (elliptic + parabolic pde, non-linear coupling via an ode system) [2] coupled with non-linear elasticity. The simulation bases on unstructured 3D meshes with anisotropic, inhomogeneous material coefficients. Besides choosing the AMG components such that the overall runtime is minimized, we needed a highly efficient MPI + OpenMP parallelization with an additional acceleration on GPUs. The presentation will focus on all the little improvements necessary to achieve very good strong speedup on 4096 CPU cores [1] such that one coupled system with 11 Mill. degrees of freedom can be solved in less than one second. The parallelization for many-core processors as NVIDA GPUs results in further speedup between 5 and 30 depending on the subtasks. In order to reduce the data transfer between accelerator memory and CPU memory in the non-linear solvers, we had to redesign the interfaces and data structures in the whole medical simulation code according to plain data structures and flexible solver steps. Although this parallelization has been done in CUDA [2] the future development will use pragma driven parallelization in OpenACC (GPU) and/or OpenMP 4.0 (Intel Xeon Phi) in order to have one code for all current (and future) many-core hardware.

Key words: many-core parallelization, algebraic multigrid, bidomain equations, elasticity
Mathematics subject classifications (2010): 65N55, 65Y10, 74-04, 65F08

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*This research was partially supported by Austrian Science Fund (FWF): F32-N18
†Institute for Mathematics and Scientific Computing, Karl Franzens University of Graz, Austria, email: gundolf.haase@uni-graz.at
‡Institute of Biophysics, Medical University of Graz, Austria

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Boundary integral formulation for the electrical response of biological cells to external electrical stimulations*

Fernando Henríquez† Carlos Jerez-Hanckes† Fernando Altermatt‡

Abstract

We present a novel formulation for modeling the electrical activity of biological cells under external electrical stimulus together with a suitable scheme to numerically solve it. Unlike previous methodologies, we are able to take into account the presence on dynamic ionic channels, cell morphology and stimulating waveforms depending on space and time. By means of boundary integral operators, we cast the original intra- and extra-cellular problem in terms of boundary quantities defined only over the cellular membrane. The numerical discretization is performed using low order basis functions whereas the time integration is performed using a semi-implicit second order accuracy method. The stability analysis suggests the existence of a maximum admissible time step depending only on problem parameters, but not on the grid size. Besides, error estimates show that if the time step is chosen proportional to the mesh size, then a rate of convergence equal to two can be achieved. Finally, we show results for cells with different shapes and stimulation waveforms. Numerical experiments validate theoretical analysis and agree with the expected biological behavior.

Key words: boundary integral equations, time-stepping schemes, Hodgkin-Huxley model, stability analysis, semi-implicit scheme

Mathematics subject classifications (2010): 35Q92, 46N60, 62P10, 92C50

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†School of Engineering, Pontificia Universidad Católica de Chile, Santiago, Chile, emails: fjhenriq@uc.cl, cjerez@ing.puc.cl

‡Department of Anesthesiology, Faculty of Medicine, Pontificia Universidad Católica de Chile, Santiago, Chile, e-mail: fernando.altermatt@gmail.com

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Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

DPG analysis: adjoint problems and test norms *

Norbert Heuer † Michael Karkulik ‡

Abstract

Standard analysis of the discontinuous Petrov-Galerkin method (DPG) with optimal test functions is based on a direct relationship between trial and test spaces, and their norms. Depending on the particular problem under consideration, theoretical and practical requirements imply different conditions both for the selection of spaces and for the definition of norms. Key ingredient is a regularity analysis of the adjoint problem. For an appropriate selection of norms, it is well posed. In this talk, we discuss several examples (like convection-dominated diffusion, non-conforming trace approximation, hypersingular boundary integral operators, and a coupled PDE-boundary integral operators scheme) and show how problem-dependent objectives force the selection of spaces and norms.

Key words: discontinuous Petrov-Galerkin method with optimal test functions, regularity analysis, adjoint problem

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†Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Avda. Vicuña Mackenna 4860, Santiago, Chile, email: `nheuer@mat.puc.cl`

‡Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Avda. Vicuña Mackenna 4860, Santiago, Chile, email: `mkarkulik@mat.puc.cl`

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Anisotropic estimates for H(**curl**)- and H(**div**)- conforming elements on prisms and applications*

Alexis Jawtuschenko† Ariel L. Lombardi‡

Abstract

The solution of the Poisson problem with Dirichlet boundary conditions on non convex polyhedral domains may show singularities on edges or vertices which degrade the convergence of the FEMs [2]. One way to recover the optimal order consists of using graded meshes, which are inevitably anisotropic. Similar behaviors exhibit the solutions of other problems as, for instance, Stokes systems and Maxwell time harmonic equations [7]. We consider the edge elements of Nédélec and the Raviart-Thomas elements [5, 6] in arbitrarily anisotropic prisms, and their corresponding k-th order interpolation operators and prove anisotropic and uniform local interpolation error estimates for each one of them. This is a generalization of what is known for tetra and hexahedra [3, 4] and, in practice, it allows to reduce the number of degrees of freedom as well as to avoid the use of a kind of narrow tetrahedra for which anisotropic error estimates are not valid. As an application, for the mixed formulation of the Poisson problem with data in L^2 on a polyhedron with concave edges, using the well known family of graded meshes [1], we prove optimal approximation error estimates. We use the fact that the regularity of the solution to the problem treated here can be characterized in terms of weighted Sobolev norms.

Key words: anisotropic finite elements, mixed finite elements, Raviart-Thomas interpolation, edge elements

Mathematics subject classifications (1991):

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†IMAS-CONICET and Departamento de Matemática, Universidad de Buenos Aires, Argentina, email: ajawtu@dm.uba.ar

‡Universidad Nacional General Sarmiento, Universidad de Buenos Aires, CONICET, e-mail: aldoc7@dm.uba.ar

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

High-order semi-Lagrangian schemes for static Hamilton-Jacobi-Bellman equations

Dante Kalise*

Abstract

A standard tool for the solution of optimal control problems is the application of the Dynamic Programming Principle proposed by Bellman in the 50's. In this context, the value function of the optimal control problem is characterized as the viscosity solution of a first-order and fully nonlinear Hamilton-Jacobi-Bellman (HJB) equation. A major advantage of the approach is the existence of a feedback mapping connecting the current state of the system and the associated optimal control. However, since the HJB equation has to be solved in a state space of the same dimension as the system dynamics, the approach is only feasible for low dimensional dynamics and it strongly relies on the use of efficient numerical approximations. In this talk, we present a high-order semi-Lagrangian scheme for the approximation of stationary HJB equations. The resulting nonlinear discrete system is solved via a fixed point approximation scheme. The convergence of the fixed point operator is justified by an ϵ-monotonicity argument.

Key words: Hamilton-Jacobi equations, fixed point approximation schemes, ϵ-monotonicity, high-order methods, semi-Lagrangian schemes
Mathematics subject classifications (1991): 65M12, 49L25, 65M06, 65M08

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*Optimization and Optimal Control Group, Johann Radon Institute for Computational and Applied Mathematics, Altenberger Straße 69, 4040 Linz, Austria. e-mail: dante.kalise@oeaw.ac.at

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Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Local high-order regularization and applications to *hp*-methods*

Michael Karkulik † Jens Markus Melenk ‡

Abstract

The regularization of a function is a basic tool in the theory of function spaces. Based on a length-scale ε, a function $u \in L_1$ is approximated by a function $u_\varepsilon \in C^\infty$, and the error $u - u_\varepsilon$ is to be quantified in terms of ε (which goes to zero). A simple form in $\mathbb{R}^d$ is the convolution with the scaled version $\rho_\varepsilon(\cdot) := \varepsilon^{-d}\rho(\cdot/\varepsilon)$ of a compactly supported smooth function ρ. The corresponding tool in numerical analysis is *quasi-interpolation.* In the h-version one constructs, e.g., from a function $u \in L_2$ a piecewise linear and globally continuous approximation on a given mesh. This is usually done by local averaging as in [1]. The link to regularization is hence to choose a spatially varying length scale $\varepsilon \sim h$, h being the (local) mesh-size. In the p or hp-version, polynomial approximations are constructed piecewise, and continuity requirements are enforced in a second step by lifting operators. Therefore, the approximated function needs to have certain regularity. This can be circumvented by patchwise constructions, cf. [2]. In this talk, we present a generalization of [1] to hp-quasi-interpolation. We construct regularization operators that are based on high-order averaging on a variable length scale. We derive simultaneous approximation properties in scales of Sobolev spaces and inverse estimates. We present two applications of this regularization.

- We link this process to local approximation orders of piecewise polynomial spaces, i.e., $\varepsilon(\cdot) \sim h/p$. In this manner, we obtain hp-quasi-interpolation operators.
- We employ the regularization operators to obtain residual a-posteriori error estimates for hp-boundary element methods.

Key words: quasi-interpolation, hp-FEM, hp-BEM

Mathematics subject classifications (1991): 65N30, 65N35, 65N50

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†Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Santiago, email: mkarkulik@mat.puc.cl

‡Institute for Analysis and Scientific Computing, Vienna University of Technology, Austria, email: melenk@tuwien.ac.at.

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A finite volume method for a two-phase multicomponent polymer flooding

SUDARSHAN K. KENETTINKARA* PRAVEEN CHANDRASHEKHAR†
VEERAPPA GOWDA ‡

Abstract

Multicomponent polymer flooding is an enhanced oil recovery technique used in petroleum industry. A simulation model of this process is governed by a system of coupled non-strictly hyperbolic conservation laws. In this talk we present a finite volume method to execute the numerical simulation of this model. In the presence of gravity, the flux functions involved in the modelling equations need not be monotone and hence designing Godunov type upwind schemes is difficult and computationally expensive. To overcome this difficulty, we use the basic idea of discontinuous flux to reduce the coupled system into an uncoupled system of scalar conservation laws with discontinuous coefficients. For these scalar equations we use the basic idea of discontinuous flux to construct a second order scheme. The scheme is shown to satisfy a maximum principle and the performance of the scheme is shown on both one and two dimensional test problems.

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*CI^{2}MA, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: `skumar@ci2ma.udec.cl`
†TIFR CAM Bangalore, India, e-mail: `praveen@math.tifrbng.res.in`
‡TIFR CAM Bangalore, India, e-mail: `gowda@math.tifrbng.res.in`

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Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Space-time adaptive multiresolution techniques with a gradient-augmented level set method for advection equations

Dmitry Kolomenskiy* Jean–Christophe Nave† Kai Schneider‡

Abstract

A space-time adaptive scheme is presented for solving advection equations in two space dimensions. Advection problems are encountered for example in moving fronts for a given velocity field, or in transport of passive scalars modeling pollution or mixing in chemical engineering. It can also be viewed as a simple model that partly describes other, more complex problems, such as advection-reaction-diffusion, fluid flow, elasticity, etc. The gradient-augmented level set method using a semi-Lagrangian formulation with backward time integration is coupled with a point value multiresolution analysis using Hermite interpolation. Thus locally refined dyadic spatial grids are introduced which are efficiently implemented with dynamic quad-tree data structures. For adaptive time integration, an embedded RungeKutta method is employed. The precision of the new fully adaptive method is analysed and speed up of CPU time and memory compression with respect to the uniform grid discretization are reported. Details can be found in [1]

Key words: Space-time adaptivity; Gradient augmented level-set method; Hermite multiresolution; Advection equation;

Mathematics subject classifications (2000): 35L65, 35Q35, 65M25, 65M50

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*CRM and The Department of Mathematics and Statistics, McGill University, 805 Sherbrooke W. Street, Montreal, QC, Canada H3A 0B9, e-mail: dkolom@gmail.com

†The Department of Mathematics and Statistics, McGill University, 805 Sherbrooke W. Street, Montreal, QC, Canada H3A 0B9, e-mail: jcnave@mcgill.edu

‡M2P2CNRS, Aix-Marseille Université 39, rue Frédéric Joliot-Curie, 13453 Marseille Cedex 13, France, email: kschneid@cmi.univ-mrs.fr

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Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Some computational techniques to improve component-wise finite-difference WENO schemes *

María del Carmen Martí[†] Pep Mulet[‡]

Abstract

This work is centered on the use of component-wise High-Resolution Shock-Capturing schemes as an alternative to the use of characteristic-wise schemes, based on the use of the spectral decomposition of the Jacobian matrix of the fluxes for upwinding that, in many cases, is not available or is quite difficult to obtain. In an attempt to improve the results obtained when using a component-wise finite-difference WENO scheme, we explore some flux-splitting schemes, such as one based on using a biased flux-splitting, named HLL flux-splitting, first introduced in [4] as a Riemann solver, that uses the estimated values of the minimum and maximum eigenvalues of the Jacobian matrix of the fluxes, instead of the spectral radius of it, as the Global Lax Friedrichs (GLF) flux-splitting schemes do. This new scheme reduces the spurious oscillations that may appear when using GLF scheme [8]. On the other hand, we analyze the use of a high-order reconstruction method with a control of the oscillations obtained by redesigning the weights used in the WENO reconstructions proposed in [5, 6, 9]. When we use those weights, introduced in [1], we reduce the oscillatory behavior while maintaining the high resolution of the scheme. Finally, we explore the use of adaptivity, as in [2, 3, 7], developing a hybrid scheme that only uses the characteristic information of the Jacobian matrix of the system in regions where singularities exist or are starting to develop, while it uses a component-wise approximation of the scheme in smooth regions, in order to speed up computing times.

Key words: finite-difference scheme, high-order reconstruction method, component-wise, Characteristic wise, flux-splitting

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†CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, Chile, email: maria.c.marti@uv.es

‡Departament de Matemàtica Aplicada, Universitat de València, València, España, e-mail: mulet@uv.es

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A discussion on the transmission conditions for saturated fluid flow through porous media with fractal microstructure*

Fernando A. Morales †

Abstract

In this talk we discuss fluid transmission conditions for the phenomenon of fluid flow through saturated porous media containing a fractal microstructure. Several models will be discussed pursuing normal stress and normal flux balance across the fractal microstructure. It will be shown that the "natural" Hilbert spaces for modeling the problem can lead to extremely trivial and fully decoupled problems, if no further considerations are done on the transmission conditions themselves. These should be closely related to the fractal microstructure. With the aforementioned considerations it will be shown how limited can be the understanding of the problem with variational formulation methods. Additionally, the graph energy model will be presented as an alternative, where it is possible to attain a more detailed description of the problem. The model is possible by introducing an "averaged" fluid transmission condition, rather that the classical pointwise balance. These "upscaled" fluid transmission condition allows to successfully couple the fractal scale (micro scale) with the flied scale (macro scale) of the domain. Time permitted, some numerical examples will be presented.

Key words: microstructure models, fractals, saturated flow through porous media.

Mathematics subject classifications (1991): 82B24, 80M40, 76S05

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†Escuela de Matemáticas, Universidad Nacional de Colombia, Sede Medellín. email: `famoralesj@unal.edu.co`

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Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Achieving robustness in domain decomposition methods

Nicole Spillane* Victorita Dolean† Patrice Hauret‡
Pierre Jolivet Frédéric Nataf§ Clemens Pechstein¶
Daniel J. Rixen‖ Robert Scheichl**

Abstract

Domain decomposition methods are a popular way to solve large linear systems. For problems arising from practical applications it is likely that the equations will have highly heterogeneous coefficients. For example a tire is made both of rubber and steel, which are two materials with very different elastic behaviour laws. Many domain decomposition methods do not perform well in this case, specially if the decomposition into subdomains does not accommodate the coefficient variations. For three popular domain decomposition methods (Additive Schwarz, BDD and FETI) we propose a remedy to this problem based on local spectral decompositions. Numerical investigations for the linear elasticity equations will confirm robustness with respect to heterogeneous coefficients, automatic (non regular) partitions into subdomains and nearly incompressible behaviour.

Key words: domain decomposition, generalized eigenvectors, guaranteed convergence

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*CMM, Universidad de Chile, Santiago, Chile, email: nspillane@dim.uchile.cl
†University of Strathclyde, Glasgow, United Kingdom
‡Michelin, Clermont Ferrand, France
§LJLL and CNRS, Université Pierre et Marie Curie, Paris, France
¶RICAM, Johannes Kepler Universitat, Linz, Austria
‖Technische Universitat Munchen, Munchen, Germany
**University of Bath, Bath, United Kingdom

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Mathematical modeling of bacterial communities: Mechanical behavior of biofilms*

Giordano Tierra[†] Juan P. Pavissich Robert Nerenberg[‡]
Zhiliang Xu and Mark S. Alber[§]

Abstract

We present an energetic variational approach to simulate biofilm interaction with fluid flow fields. It is aimed to model biofilm heterogeneity and constitutive behaviour through a formulation that describes multicomponent complex fluids with viscoelastic properties. The biofilm is assumed an incompressible fluid made of a ternary mixture of bacteria, extracellular polymeric substance (EPS), and solvent phases. Biofilm response to mechanical stress is incorporated through cohesion and elastic energy functionals. Our model satisfies an overall dissipative energy law and is solved with an efficient unconditionally energy-stable splitting scheme. Numerical simulations illustrate that the model is able to capture large-scale events of biofilm deformation and detachment under different hydrodynamic conditions in elastic and viscous regimes. The results are qualitatively consistent with laboratory experiments. Simulations show the effect of mechanical parameters and the role of the EPS in the hydrodynamic interaction with flow fields. Higher viscosity provides stronger resistance to deformation, and longer elastic relaxation can lead to the formation of biofilm filaments. This formulation and the numerical algorithm developed provide a robust mathematical and physical framework to study biofilm mechanical behaviour, considering structural heterogeneity and complex rheology.

Key words: biofilm, viscoelasticity, phase-field, energy stability, continuum mechanics, splitting schemes

Mathematics subject classifications (1991): 65M60, 76D05, 76T30, 92B05

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[†]Mathematical Institute, Faculty of Mathematics and Physics, Charles University, 186 75 Prague 8, Czech Republic, email: `gtierra@karlin.mff.cuni.cz`

[‡]Department of Civil and Environmental Engineering and Earth Sciences, University of Notre Dame, Notre Dame, IN 46556, USA, e-mail: {`jpavissi,nerenberg.1`}`@nd.edu`

[§]Department of Applied and Computational Mathematics and Statistics, University of Notre Dame, Notre Dame, IN 46556, USA, e-mail: {`zxu2,malber`}`@nd.edu`

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