LA SERENA NUMERICA II

Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales

Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

A priori error analysis for HDG methods in curved domains using extensions from polyhedral subdomains

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Abstract

We present an a priori error analysis of a technique ([3], [4]) that allows us to numerically solve diffusion problems with Dirichlet boundary conditions defined on curved domains Ω by using finite element methods defined in polyhedral subdomains $\mathsf{D}_h \subseteq \Omega$. We prove that the order of convergence in the L^2-norm of the approximate flux and scalar unknowns is optimal as long as the distance between the boundary of the original domain Γ and that of the computational domain Γ_h is of order h. We also prove that the L^2-norm of a projection of the error of the scalar variable superconverges with a full additional order when the distance between Γ and Γ_h is of order $h^{5/4}$ but with only half an additional order when such a distance is of order h. In addition, we present numerical experiments validating the theoretical results and showing that even when the distance between Γ and Γ_h is of order h, the above-mentioned projection of the error of the scalar variable can still superconverge with a full additional order.

Key words: HDG, curved domains, immerse boundary method

Mathematics subject classifications (2010): 65N30, 65M60

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