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Analysis of the HDG method for the Stokes-Darcy coupling*

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Abstract

In this talk we introduce and analyze a hybridizable discontinuous Galerkin (HDG) method for numerically solving the coupling of fluid flow with porous media flow. Flows are governed by the Stokes and Darcy equations, respectively, and the corresponding transmission conditions are given by mass conservation, balance of normal forces, and the Beavers-Joseph-Saffman law. We consider a fully-mixed formulation in which the main unknowns are given by the stress, the vorticity, the velocity, and the trace of the velocity, all them in the fluid, together with the velocity, the pressure, and the trace of the pressure in the porous medium. In addition, we enrich the finite dimensional subspace for the stress, in order to obtain optimally convergent approximations for all unknowns, as well as a superconvergent approximation of the trace variables. To do that, similarly as in previous papers dealing with development of the *a priori* error estimates, we use the projection-based error analysis in order to simplify the corresponding study. Finally, we provide several numerical results illustrating the good performance of the proposed scheme and confirming the optimal order of convergence provided by the HDG approximation.

Key words: coupling, Stokes equations, Darcy equations, mixed finite element method, hybridized discontinuous Galerkin method

Mathematics subject classifications (2012): 65N15, 65N30, 35L65, 74F10, 74S05

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