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# High order boundary extrapolation techniques for finite difference WENO schemes on complex domains *

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**Abstract**

Finite difference WENO schemes [7, 9] have become an efficient method for the approximate solution of multidimensional hyperbolic conservation laws. These schemes follow a method of lines strategy, for which the spatial discretization is obtained by numerical differentiation of reconstruction of fluxes. Higher order accuracy can then be obtained from highly accurate reconstructions and these can be quite readily designed as long as the underlying mesh is an equispaced Cartesian mesh. In this context, the application of suitable numerical boundary conditions for hyperbolic conservation laws on domains with complex geometry has become a problem with certain difficulty that has been tackled in different ways according to the nature of the numerical methods and mesh type([3, 10, 11, 12]. In this presentation, we propose an extrapolation technique on structured Cartesian meshes (which, as opposed to non-structured meshes, can not be adapted to the morphology of the domain boundary) of the information in the interior of the computational domain to ghost cells. This technique is based on the application of Lagrange interpolation with a previous filter for the detection of discontinuities that permits a data dependent extrapolation, with higher order at smooth regions and essentially non oscillatory properties near discontinuities. Some simulations with the Euler equations of gas dynamics on relatively complex domains ([13, 3]) are presented in order to assess the capabilities of the proposed techniques.

**Key words**: Hyperbolic conservation laws, finite difference WENO schemes, boundary extrapolation, Euler equations.

**Mathematics subject classifications (1991)**: 65N06, 65N40, 65N50

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