## LA SERENA NUMERICA II

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## A discussion on the transmission conditions for saturated fluid flow through porous media with fractal microstructure<sup>\*</sup>

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## Abstract

In this talk we discuss fluid transmission conditions for the phenomenon of fluid flow through saturated porous media containing a fractal microstructure. Several models will be discussed pursuing normal stress and normal flux balance across the fractal microstructure. It will be shown that the "natural" Hilbert spaces for modeling the problem can lead to extremely trivial and fully decoupled problems, if no further considerations are done on the transmission conditions themselves. These should be closely related to the fractal microstructure. With the aforementioned considerations it will be shown how limited can be the understanding of the problem with variational formulation methods. Additionally, the graph energy model will be presented as an alternative, where it is possible to attain a more detailed description of the problem. The model is possible by introducing an "averaged" fluid transmission condition, rather that the classical pointwise balance. These "upscaled" fluid transmission condition allows to successfully couple the fractal scale (micro scale) with the flied scale (macro scale) of the domain. Time permitted, some numerical examples will be presented.

Key words: microstructure models, fractals, saturated flow through porous media.

Mathematics subject classifications (1991): 82B24, 80M40, 76S05

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