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Some computational techniques to improve component-wise finite-difference WENO schemes *

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Abstract

This work is centered on the use of component-wise High-Resolution Shock-Capturing schemes as an alternative to the use of characteristic-wise schemes, based on the use of the spectral decomposition of the Jacobian matrix of the fluxes for upwinding that, in many cases, is not available or is quite difficult to obtain. In an attempt to improve the results obtained when using a component-wise finite-difference WENO scheme, we explore some flux-splitting schemes, such as one based on using a biased flux-splitting, named HLL flux-splitting, first introduced in [4] as a Riemann solver, that uses the estimated values of the minimum and maximum eigenvalues of the Jacobian matrix of the fluxes, instead of the spectral radius of it, as the Global Lax Friedrichs (GLF) flux-splitting schemes do. This new scheme reduces the spurious oscillations that may appear when using GLF scheme [8]. On the other hand, we analyze the use of a high-order reconstruction method with a control of the oscillations obtained by redesigning the weights used in the WENO reconstructions proposed in [5, 6, 9]. When we use those weights, introduced in [1], we reduce the oscillatory behavior while maintaining the high resolution of the scheme. Finally, we explore the use of adaptivity, as in [2, 3, 7], developing a hybrid scheme that only uses the characteristic information of the Jacobian matrix of the system in regions where singularities exist or are starting to develop, while it uses a component-wise approximation of the scheme in smooth regions, in order to speed up computing times.

Key words: finite-difference scheme, high-order reconstruction method, component-wise, Characteristic wise, flux-splitting

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