LA SERENA NUMERICA II
Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Local high-order regularization and applications to *hp*-methods*

Michael Karkulik † Jens Markus Melenk ‡

Abstract

The regularization of a function is a basic tool in the theory of function spaces. Based on a length-scale ε, a function $u \in L_1$ is approximated by a function $u_\varepsilon \in C^\infty$, and the error $u - u_\varepsilon$ is to be quantified in terms of ε (which goes to zero). A simple form in $\mathbb{R}^d$ is the convolution with the scaled version $\rho_\varepsilon(\cdot) := \varepsilon^{-d}\rho(\cdot/\varepsilon)$ of a compactly supported smooth function ρ. The corresponding tool in numerical analysis is *quasi-interpolation*. In the h-version one constructs, e.g., from a function $u \in L_2$ a piecewise linear and globally continuous approximation on a given mesh. This is usually done by local averaging as in [1]. The link to regularization is hence to choose a spatially varying length scale $\varepsilon \sim h$, h being the (local) mesh-size. In the p or hp-version, polynomial approximations are constructed piecewise, and continuity requirements are enforced in a second step by lifting operators. Therefore, the approximated function needs to have certain regularity. This can be circumvented by patchwise constructions, cf. [2]. In this talk, we present a generalization of [1] to hp-quasi-interpolation. We construct regularization operators that are based on high-order averaging on a variable length scale. We derive simultaneous approximation properties in scales of Sobolev spaces and inverse estimates. We present two applications of this regularization.

- We link this process to local approximation orders of piecewise polynomial spaces, i.e., $\varepsilon(\cdot) \sim h/p$. In this manner, we obtain hp-quasi-interpolation operators.
- We employ the regularization operators to obtain residual a-posteriori error estimates for hp-boundary element methods.

Key words: quasi-interpolation, hp-FEM, hp-BEM

Mathematics subject classifications (1991): 65N30, 65N35, 65N50

*This research was partially supported by CONICYT-Chile through FONDECYT project 3140614.

†Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Santiago, email: mkarkulik@mat.puc.cl

‡Institute for Analysis and Scientific Computing, Vienna University of Technology, Austria, email: melenk@tuwien.ac.at.

References

[1] Clément, P., *Approximation by finite element functions using local regularization.* Rev. Française Automat. Informat. Recherche Opérationnelle Sér. RAIRO Analyse Numérique, 9(R-2):77–84, 1975.

[2] Melenk, J.M., *hp-interpolation of nonsmooth functions and an application to hp-a posteriori error estimation.* SIAM J. Numer. Anal., 43(1):127–155, 2005.