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Boundary integral formulation for the electrical response of biological cells to external electrical stimulations*

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Abstract

We present a novel formulation for modeling the electrical activity of biological cells under external electrical stimulus together with a suitable scheme to numerically solve it. Unlike previous methodologies, we are able to take into account the presence on dynamic ionic channels, cell morphology and stimulating waveforms depending on space and time. By means of boundary integral operators, we cast the original intra- and extra-cellular problem in terms of boundary quantities defined only over the cellular membrane. The numerical discretization is performed using low order basis functions whereas the time integration is performed using a semi-implicit second order accuracy method. The stability analysis suggests the existence of a maximum admissible time step depending only on problem parameters, but not on the grid size. Besides, error estimates show that if the time step is chosen proportional to the mesh size, then a rate of convergence equal to two can be achieved. Finally, we show results for cells with different shapes and stimulation waveforms. Numerical experiments validate theoretical analysis and agree with the expected biological behavior.

Key words: boundary integral equations, time-stepping schemes, Hodgkin-Huxley model, stability analysis, semi-implicit scheme

Mathematics subject classifications (2010): 35Q92, 46N60, 62P10, 92C50

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