LA SERENA NUMERICA II
**Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales**
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

# Multilevel methods for the hypersingular integral equation on locally refined triangulations*

Michael Feischl[†] Thomas Führer[‡]
Dirk Praetorius[§] Ernst P. Stephan[¶]

**Abstract**

We consider the hypersingular integral equation for the 2D and 3D Laplacian. It is well-known that the condition number of the Galerkin matrix grows with the number of elements as well as the global mesh-size quotient $h_{\max}/h_{\min}$ as the mesh is (locally) refined. Therefore, the developement of *optimal* preconditioners is a necessary and important task. Here, optimality is understood in the sense that the resulting condition numbers are independent of the number of elements and the mesh-size. In this talk, we present results from [1], where we consider a (local) multilevel diagonal preconditioner. The basic idea of this preconditioner is to consider only newly created nodes in $\mathcal{T}_{\ell+1}\backslash\mathcal{T}_\ell$ plus some of their immediate neighbours for preconditioning. For uniform refinement, it was proved in [3] that multilevel diagonal preconditioners are optimal. On locally refined triangulations such a result was unknown. Basically, the proof of optimality consists of providing a stable subspace decomposition for the fractional order Sobolev space $H^{1/2}$ in the frame of additive Schwarz methods. Our analysis relies on an appropriate variant of the Scott-Zhang projection [2] and hierarchical properties of the mesh-refinement employed. Numerical examples on closed and open boundaries underline our theoretical results.

**Key words**: adaptivity, multilevel methods, hypersingular integral equation

**Mathematics subject classifications (2010)**: 65N30, 65F08, 65N38

*This research was partially supported through the FWF research project*Adaptive Boundary Element Method*, see `http://www.asc.tuwien.ac.at/abem/`, funded by the Austrian Science Fund (FWF) under grant P21732, as well as through the *Innovative Projects Initiative* of Vienna University of Technology. This support is thankfully acknowledged.

[†]Institute for Analysis and Scientific Computing, Vienna University of Technology, Vienna, Austria, e-mail: `michael.feischl@tuwien.ac.at`

[‡]Facultad de Matemáticas, Pontificia Universidad Católica de Chile, Santiago, Chile.

[§]Institute for Analysis and Scientific Computing, Vienna University of Technology, Vienna, Austria, e-mail: `dirk.praetorius@tuwien.ac.at`

[¶]Institute for Applied Mathematics, Leibniz University Hannover, Hannover, Germany, e-mail: `stephan@ifam.uni-hannover.de`.

## References


[1] M. Feischl, T. Führer, D. Praetorius, E.P. Stephan, *Optimal multilevel additive Schwarz preconditioning for hypersingular integral equations on locally refined trinangulations*. Submitted (2014).

[2] L.R. Scott, S. Zhang, *Finite element interpolation of nonsmooth functions satisfying boundary equations*. Math. Comp. (1990), 54:483–493.

[3] T. Tran, E.P. Stephan, *Additive Schwarz methods for the h-version boundary element method*. Appl. Anal. (1996), 60:63–84.