LA SERENA NUMERICA II
Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Linearly implicit IMEX Runge-Kutta methods for a class of degenerate convection-diffusion problems*

Sebastiano Boscarino† Raimund Bürger‡ Pep Mulet§
Giovanni Russo¶ Luis M. Villada‖

Abstract

Multi-species kinematic flow models with strongly degenerate diffusive corrections give rise to systems of nonlinear convection-diffusion equations of arbitrary size. Applications of these systems include models of polydisperse sedimentation and multi-class traffic flow. Implicit-explicit (IMEX) Runge-Kutta (RK) methods [1] are suitable for the solution of these convection-diffusion problems since the stability restrictions, coming from the explicitly treated convective part, are much less severe than those that would be deduced from an explicit treatment of the diffusive term. These schemes usually combine an explicit Runge-Kutta scheme for the time integration of the convective part with a diagonally implicit one for the diffusive part. In [4] a scheme of this type is proposed, where the nonlinear and non-smooth systems of algebraic equations arising in the implicit treatment of the degenerate diffusive part are solved by smoothing of the diffusion coefficients combined with a Newton-Raphson method with line search. This nonlinearly implicit method is robust but associated with considerable effort of implementation and possibly CPU time. To overcome these shortcomings while keeping the advantageous stability properties of IMEX-RK methods, a second variant of these methods is proposed, in which the diffusion terms are discretized in a way that more carefully distinguishes between stiff and nonstiff dependence [3], such that in each time step only a linear system needs to be solved still maintaining high order accuracy in time, which makes these methods much simpler to implement. In a series of examples

*This research was partially supported by CONICYT-Chile through BASAL project CMM, Universidad de Chile; project Anillo ACT1118 (ANANUM); and by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción; Red Doctoral REDOC.CTA, MINEDUC project UCO1202; Centro CRHIAM Proyecto Conicyt Fondap 15130015, and Fondecyt projects 1130154 and

†Department of Mathematics and Computer Science, University of Catania, 95125 Catania, Italy. E-Mail: boscarino@dmi.unict.it

‡CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile, email: rburger@ing-mat.udec.cl

§Departament de Matemàtica Aplicada, Universitat de València, Av. Dr. Moliner 50, E-46100 Burjassot, Spain. E-Mail: mulet@uv.es

¶Department of Mathematics and Computer Science, University of Catania, 95125 Catania, Italy. E-Mail: russo@dmi.unict.it

‖Departamento de Matemática, Facultad de Ciencias, Universidad del Bío-Bío, Casilla 5-C, Concepción, Chile. E-Mail: lvillada@ubiobio.cl

of polydisperse sedimentation [2] and multi-class traffic flow [5] it is demonstrated that these new linearly implicit IMEX-RK schemes approximate the same solutions as the nonlinearly implicit versions, and in many cases these schemes are more efficient.

Key words: Implicit-explicit Runge-Kutta schemes, degenerate convection-diffusion equations, linearly implicit methods, polydisperse sedimentation, multiclass traffic flow.

References

[1] Ascher, U., Ruuth, S. and Spiteri, J., *Implicit-explicit Runge-Kutta methods for time dependent partial differential equations,* Appl. Numer. Math., 25 (1997), pp. 151–167.

[2] Berres, S., Bürger, R., Karlsen, K.H. and Tory, E.M., *Strongly degenerate parabolic-hyperbolic systems modeling polydisperse sedimentation with compression*, SIAM J. Appl. Math., 64 (2003), pp. 41–80.

[3] Boscarino, S., LeFloch, P.G. and Russo, G. *High order asymptotic-preserving methods for fully nonlinear relaxation problems*, SIAM J. Sci. Comput., 36 (2014), pp. A377–A395.

[4] Bürger, R., Mulet, P. and Villada, L.M., *Regularized nonlinear solvers for IMEX methods applied to diffusively corrected multi-species kinematic flow models*, SIAM J. Sci. Comput., 35 (2013), pp. B751–B777.

[5] Bürger, R., Mulet, P. and Villada, L.M., *A diffusively corrected multiclass Lighthill-Whitham-Richards traffic model with anticipation lengths and reaction times*, Adv. Appl. Math. Mech., 5 (2013), pp. 728–758.