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Mixed-primal finite element approximation of a steady sedimentation-consolidation system*

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Abstract

This work is devoted to the mathematical and numerical analysis of a coupled flow and transport system typically encountered in continuum-based models of sedimentation-consolidation processes. These problems are relevant to a variety of applications including fluidized beds, clot formation within the blood, solid-liquid separation in wastewater treatement, and many others. The model problem focuses on the steady-state regime of the phenomenon and so the governing equations consist in the Brinkman equations with variable viscosity, written in terms of Cauchy stresses and bulk velocity of the mixture; coupled with a nonlinear advection – nonlinear diffusion equation describing the transport of the volume fraction. The variational approach is based on an augmented mixed formulation for the Brinkman problem and a primal weak form for the transport equation. Solvability of the coupling is established using classical fixed-point arguments, and we introduce an augmented mixed-primal Galerkin scheme based on Raviart-Thomas approximations of order k for the stress and piecewise continuous polynomials of order $k+1$ for velocity and volume fraction. We prove existence and uniqueness of the discrete problem also based on a fixed-point strategy, and we rigorously derive optimal error estimates in the natural norms. A few numerical tests illustrate the accuracy of the augmented mixed-primal finite element method, and the properties of the model.

Key words: Brinkman equations, nonlinear transport problem, augmented mixed–primal formulation, fixed point theory, thermal convection, sedimentation-consolidation process, finite element methods, a priori error analysis.

Mathematics subject classifications (2000): 65N30, 65N12, 76R05, 76D07, 65N15.

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