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Mathematical modeling of bacterial communities: Mechanical behavior of biofilms*

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Abstract

We present an energetic variational approach to simulate biofilm interaction with fluid flow fields. It is aimed to model biofilm heterogeneity and constitutive behaviour through a formulation that describes multicomponent complex fluids with viscoelastic properties. The biofilm is assumed an incompressible fluid made of a ternary mixture of bacteria, extracellular polymeric substance (EPS), and solvent phases. Biofilm response to mechanical stress is incorporated through cohesion and elastic energy functionals. Our model satisfies an overall dissipative energy law and is solved with an efficient unconditionally energy-stable splitting scheme. Numerical simulations illustrate that the model is able to capture large-scale events of biofilm deformation and detachment under different hydrodynamic conditions in elastic and viscous regimes. The results are qualitatively consistent with laboratory experiments. Simulations show the effect of mechanical parameters and the role of the EPS in the hydrodynamic interaction with flow fields. Higher viscosity provides stronger resistance to deformation, and longer elastic relaxation can lead to the formation of biofilm filaments. This formulation and the numerical algorithm developed provide a robust mathematical and physical framework to study biofilm mechanical behaviour, considering structural heterogeneity and complex rheology.

Key words: biofilm, viscoelasticity, phase-field, energy stability, continuum mechanics, splitting schemes

Mathematics subject classifications (1991): 65M60, 76D05, 76T30, 92B05

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