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Reconstruction-based a posteriori error estimation for the coupled Stokes-Darcy problem*

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Abstract

We derive a posteriori error estimates for the mixed finite element approximations for the coupled Stokes–Darcy problem. The transmission conditions are given by the conservation of mass, the balance of normal forces, and the Beavers-Joseph-Saffman law. In particular, the normal continuity of the velocity field on the interface between fluid flow and porous media flow is imposed strongly in the space for velocity field. For the weak imposition of this condition, it has been shown that combination of any pair of stable finite element methods for the pure Stokes and pure Darcy problems yields a convergent scheme for the coupled problem in Gatica et al. (2011) [6]. For the strong incorporation of that condition, the result has been extended in Márquez et al. (2014) [8]. Our analysis is based on a conforming velocity reconstruction and locally conservative flux reconstruction. The velocity reconstruction is defined to satisfy the normal continuity condition across the interface. The flux reconstruction is defined via mixed finite element approximations of local Neumann or Neumann–Dirichlet problems following an approach in Hannukainen et al. (2012) [7]. The derived a posteriori error estimates are proven to be reliable.

Key words: a posteriori error estimate, velocity and flux reconstruction, coupled Stokes–Darcy flow

Mathematics subject classifications (1991): 65N15, 65N30, 76D07

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