LA SERENA NUMERICA II
Octavo Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
Departamento de Matemáticas, Universidad de La Serena, La Serena, Chile, Enero 14 - 16, 2015

Modelling, analysis, and numerical approximation of cardiac electromechanical interactions*

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Abstract

We present an overview of the numerical simulation of the interaction between cardiac electrophysiology, sub-cellular activation mechanisms, and macroscopic tissue contraction; that together comprise the essential elements of the electromechanical function of the human heart. We discuss the development of accurate mathematical models tailored for the simulation of the cardiac excitation-contraction mechanisms, which are primarily based on nonlinear elasticity theory and phenomenological descriptions of the mechano-electrical feedback. Here the link between contraction and the biochemical reactions at microscales is described by an active strain decomposition model. Then we turn to the mathematical analysis of a simplified version of the model problem consisting in a reaction-diffusion system governing the dynamics of ionic quantities, intra and extra-cellular potentials, and the elastodynamics equations describing the motion of an incompressible material. Under the assumption of linearized elastic behavior and a truncation of the updated nonlinear diffusivities, we are able to prove existence of weak solutions to the underlying coupled system and uniqueness of regular solutions. The proof of existence is based on a combination of parabolic regularization, the Faedo-Galerkin method, and the monotonicity-compactness method of J.L. Lions. A finite element formulation is also introduced, for which we establish existence of discrete solutions and show convergence to a weak solution of the original problem. We close with some numerical examples illustrating the convergence of the method and some features of the model.

*This research was partially supported by the ERC through the advanced grant "MATHCARD, Mathematical Modelling and Simulation of the Cardiovascular System".

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Key words: Electromechanical coupling, bidomain equations, finite element methods, coupled multiphysics, active strain, weak compactness method

Mathematics subject classifications (1991): 74F99, 35K57, 92C10, 65M60

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