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A Darcy-Brinkman-Forchheimer porous model for alloys convective-diffusion solidification in molds*

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Abstract

Two-dimensional unsteady fluid mechanics and heat transfer in liquid-solid phase changes are traditionally described by a system of five strongly coupled non-linear PDEs in the classical mathematical models. The use of the Darcy-Brinkman-Forchheimer porous model is presented in this talk to describe the fluid flow in the mushy zone. Temperature variable porosity and permeability in the mushy zone are incorporated into the macroscopic model to include relevant physical information found at the micro-scale level. We consider the solidification of an aluminum-silica alloy inside a thick walled mold. Unsteady heat conduction in the graphite mold coupled to diffusion in the solidified alloy and to the transient fluid mechanics and heat transport by convection and diffusion in the liquid phase and in the mushy region is described. Phase-change for binary Al-1.7Si alloy occurs between 650 and 550°C. Initial temperature of the melted alloy is 860°C while the mold is initially a 300°C, with convective cooling to the external air at 25°C. Boundary conditions of the third class (Robin type) are imposed on three mold walls and one of the Neumann type for the adiabatic bottom. The solution is obtained by using the PSIMPLER algorithm and the Finite Volume Method. Numerical experiments are performed to assess the effect of the convection in solidification processes with Rayleigh numbers Ra $= 10^2$, 10^3, 10^4 and 10^5. Results for the history of streamlines and isotherms calculated by the DBF model are compared with the solution obtained with the classical non-porous model. The DBF model incorporates new physical information by the additional terms, contributes to stabilize the numerical solution and reduces the CPU time needed to solve the problem.

Key words: Porous model, alloy solidification, finite volume method, sequential algorithm.

Mathematics subject classifications (2010): 80A22, 35K20, 65M60, 65N30, 76S05.

References

[1] Celentano, D., Cruchaga, M., Moraga, N., and Fuentes, J., *Modeling natural convection with solidification in mold cavities.* Numerical Heat Transfer, Part A, vol. 39, pp. 631-654, (2001).

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[2] Moraga, N., Ramirez, S., Godoy, M., and Ticchione, P., *Study of convective non-Newtonian alloy solidification in molds by the PSIMPLER/Finite Volume Method.* Numerical Heat Transfer, Part A, vol. 12, pp. 936-953, (2010).

[3] Stefanescu, D.M., *Science and Engineering of Casting Solidification.* 2^{nd} ed., Springer, Columbus, 2010.

[4] Liu, S., Li, Y., and Zhang, Y., *Mathematical solutions and numerical models employed for the investigation of PCMs phase transformations*, Renewable and Sustainable Energy Reviews, vol. 33, pp. 659-674, (2014).

[5] Yamaguchi, m., and beckermann, c., *Direct numerical simulation of solid deformation during dendritic solidification*, JOM, vol. 66, pp. 1431-1438, (2014).