A PRIORI AND A POSTERIORI ERROR ANALYSIS FOR A VORTICITY-BASED MIXED FORMULATION OF THE GENERALIZED STOKES EQUATIONS

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Abstract. This paper deals with the analysis of new mixed finite element methods for the generalized Stokes problem formulated in terms of velocity, vorticity and pressure, with non-standard boundary conditions. By employing an extension of the Babuška-Brezzi theory, it is proved that the resulting continuous and discrete variational formulations are well-posed. In particular, on the one hand we show that Raviart-Thomas elements of order $k \geq 0$ for the approximation of the velocity field, piecewise continuous polynomials of degree $k+1$ for the vorticity, and piecewise polynomials of degree k for the pressure, yield unique solvability of the discrete problem. On the other hand, we also show that families of finite elements based on Brezzi-Douglas-Marini elements of order $k+1$ for the approximation of velocity, piecewise continuous polynomials of degree $k+2$ for the vorticity, and piecewise polynomials of degree k for the pressure ensure the well-posedness of the associated Galerkin scheme. We note that these families provide exactly divergence-free approximations of the velocity field. We establish a priori error estimates in the natural norms and we carry out the reliability and efficiency analysis of a residual-based a posteriori error estimator. Finally, we report several numerical experiments illustrating the behavior of the proposed schemes and confirming our theoretical results on unstructured meshes. Additional examples of cases not covered by our theory are also presented.

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