AN AUGMENTED MIXED–PRIMAL FINITE ELEMENT METHOD FOR A COUPLED FLOW–TRANSPORT PROBLEM

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Abstract. In this paper we analyze the coupling of the Stokes equations with variable viscosity and a nonlinear convection-diffusion problem modeling the steady state of a sedimentation-consolidation process. An augmented variational approach for the fluid flow coupled with a primal formulation for the transport model is proposed. The resulting Galerkin scheme yields an augmented mixed-primal finite element method employing Raviart-Thomas spaces of order k for the Cauchy stress, and continuous piecewise polynomials of degree $\leq k+1$ for the velocity and the local solids concentration. The classical Schauder and Brouwer fixed point theorems are utilized to establish existence of solution of the continuous and discrete formulations, respectively. Then, sufficiently small data allow to prove uniqueness and to derive optimal a priori error estimates. Finally, several numerical examples illustrating the performance of the proposed method and confirming the predicted rates of convergence are reported.

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