Physical Considerations on the Schelling model of Social Segregation

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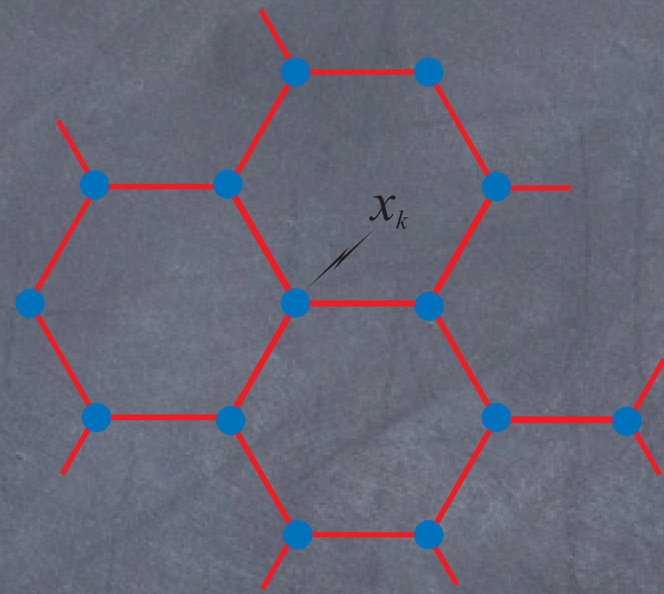
Plan

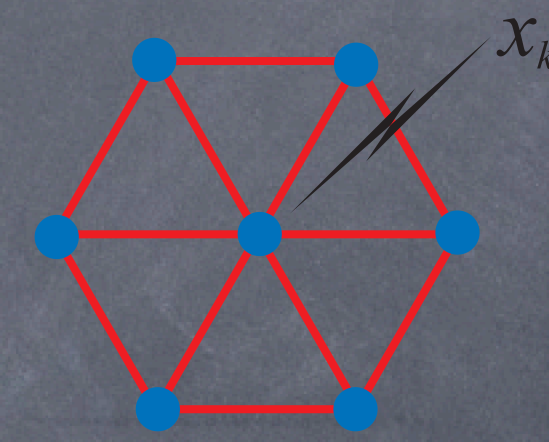
- Introduction
- The Schelling Model
- Qualitative behavior
- Quantitative behavior
- Discussion

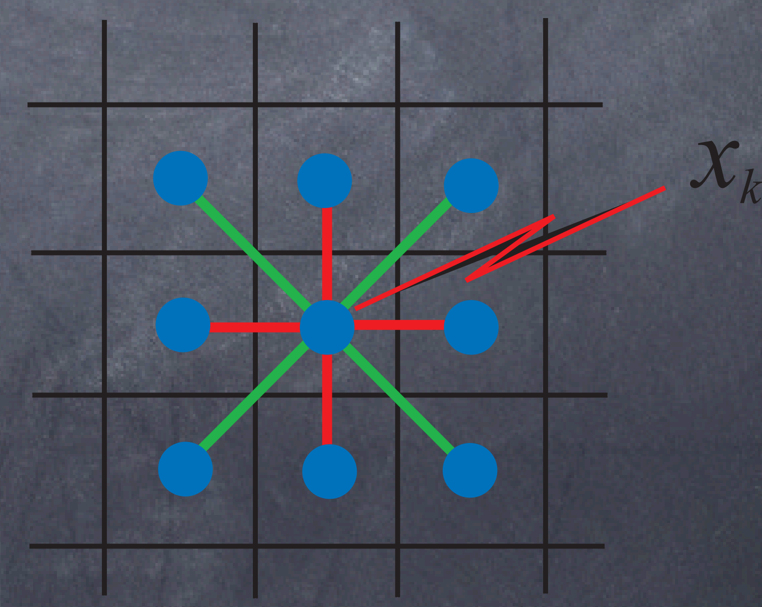
The Model of Segregation by Shelling

Thomas C. Schelling (1969 - 1972)

- Lattice {i,k}
- State $x_k = \pm 1$
- Vicinity
- Tolerance threshold



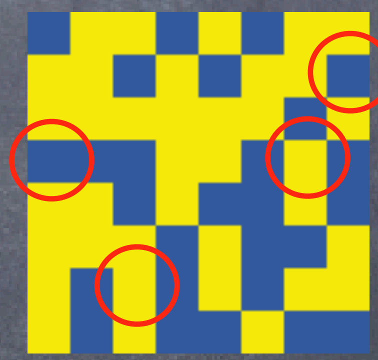




Happiness threshold

An individual is unhappy if there are more than θ individuals of the other type.

eg. in a vicinity of 8 neighbors and if $\theta = 5$ then :



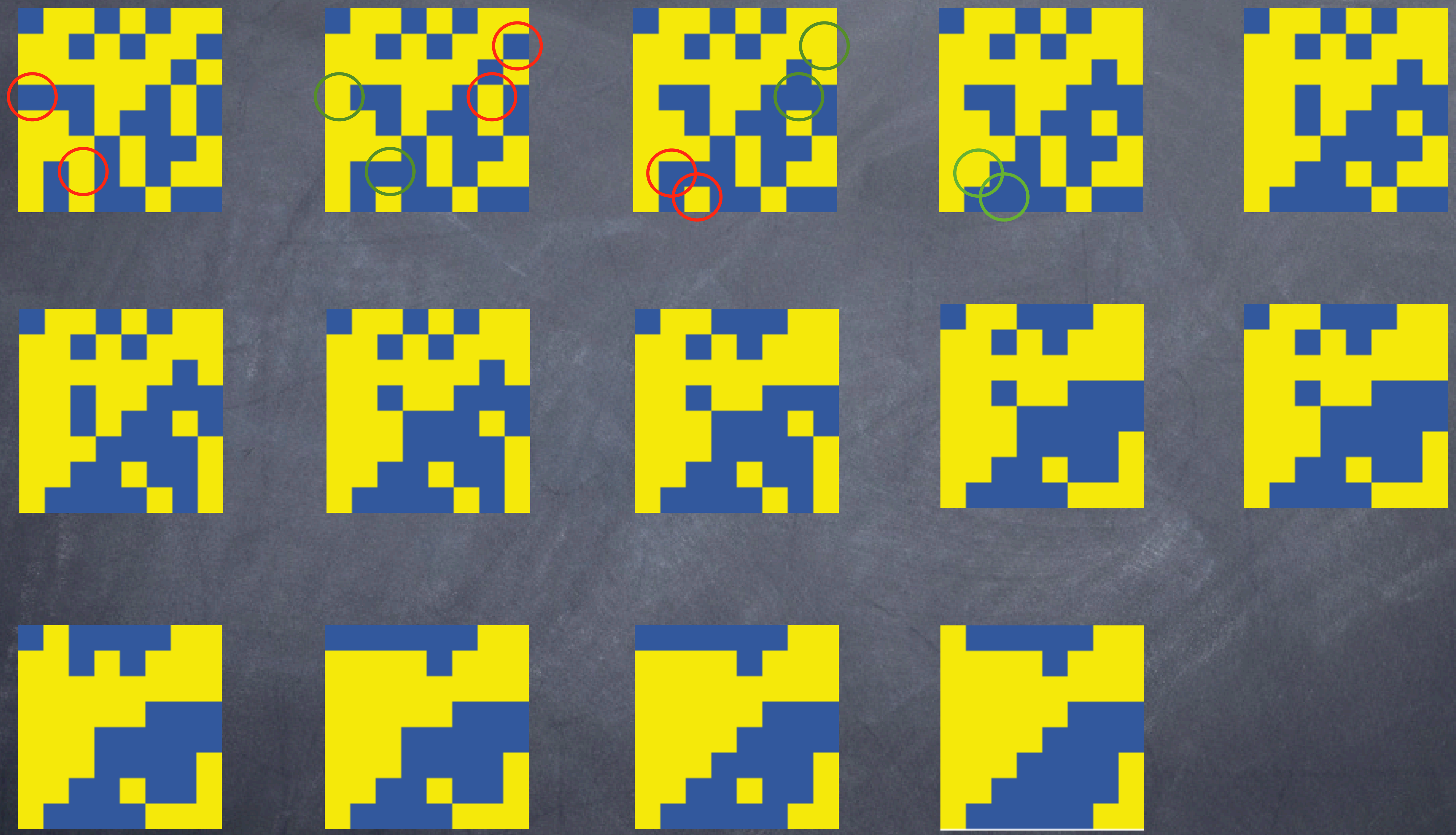
The rule

At each step, one lists the unhappy individuals of both species, and then randomly one exchanges two individuals of opposite specie.

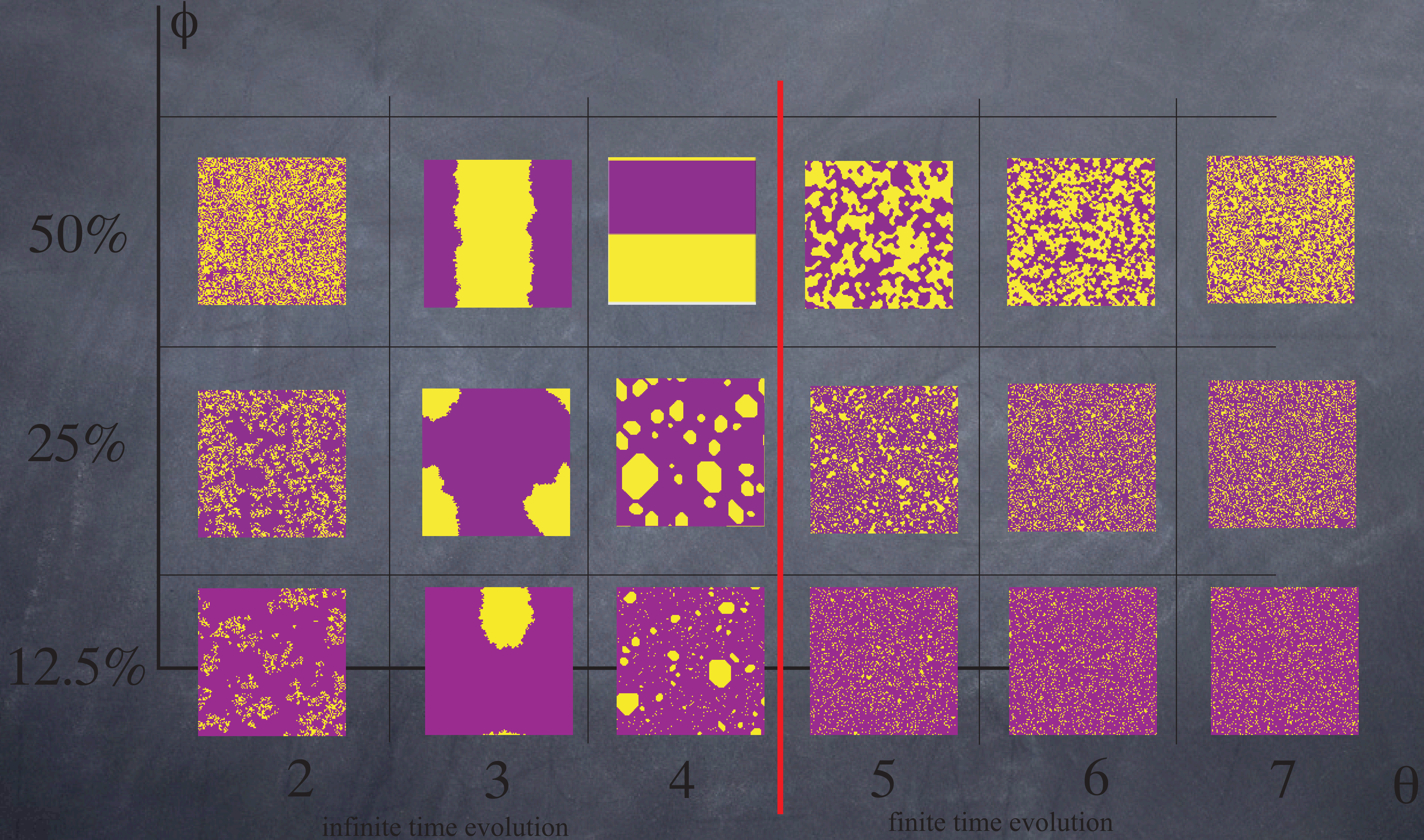
Remark: the number of individuals of each specie (N_+ & N_-) are conserved.

$$\sum_{i,k} x_{ik} = N_+ - N_- = Cte$$

$$\theta = 5$$



Phase diagram

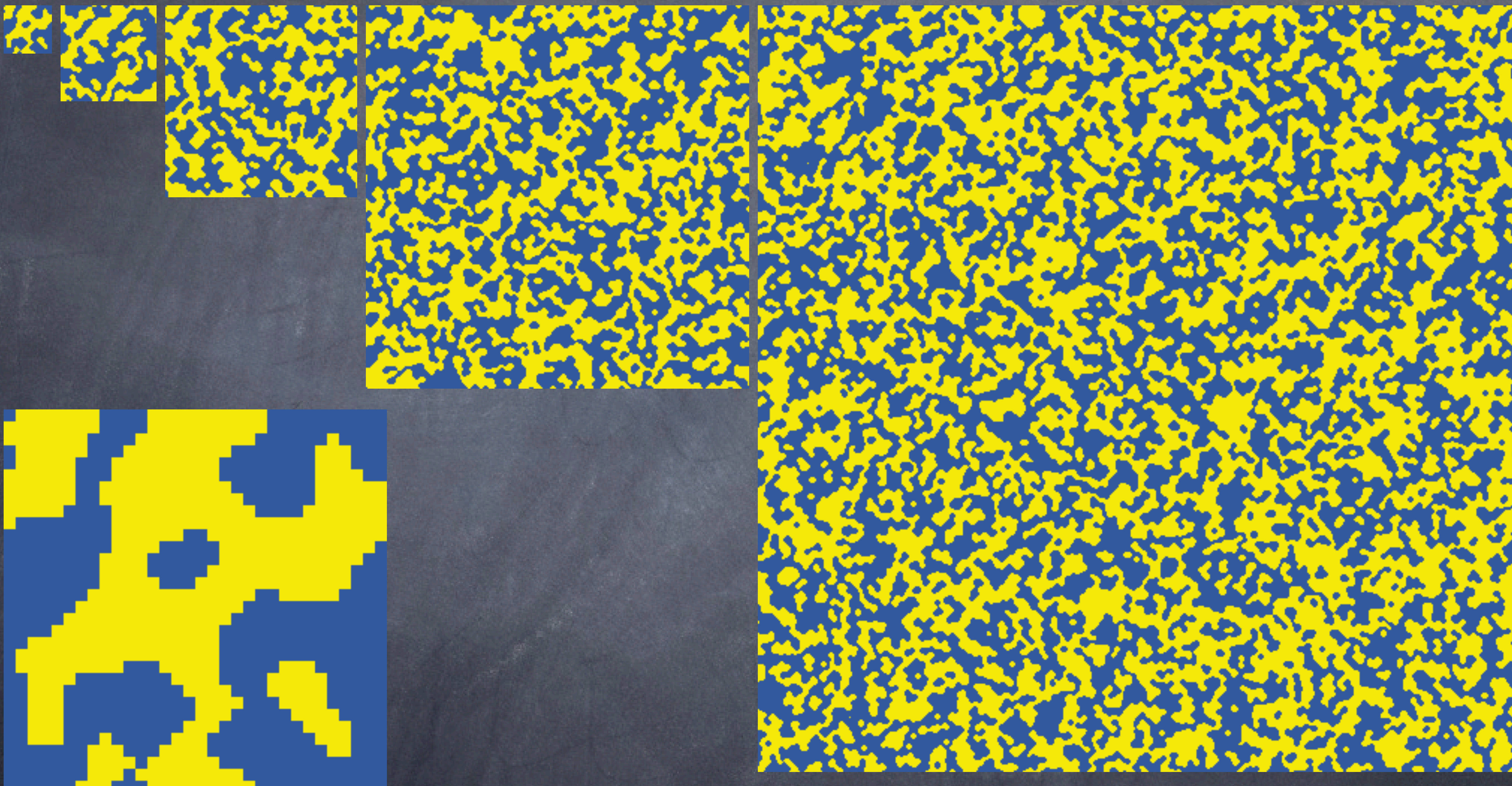


$$\phi = \frac{N_+}{N_+ + N_-}$$

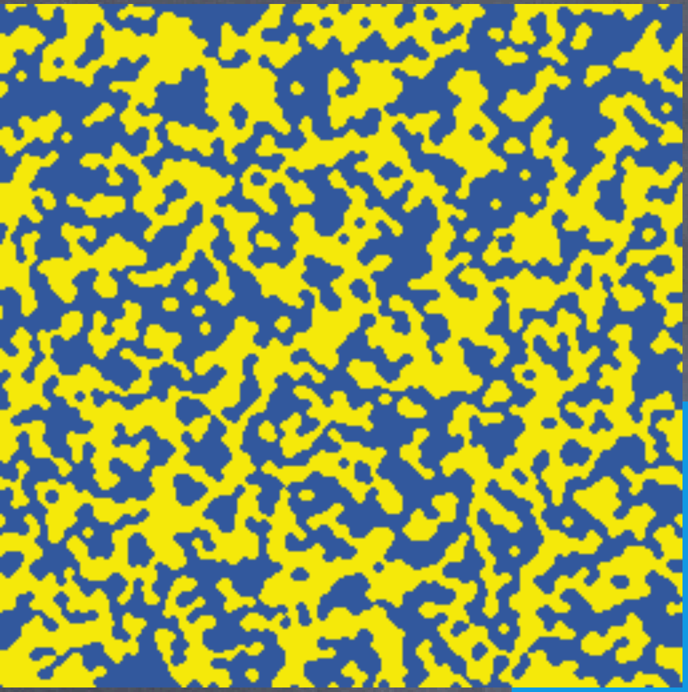
Comments

- A tendency of segregation.
- A tendency of a diminution of the interfaces
- But! there is a strong frustration.
- A length scale ?

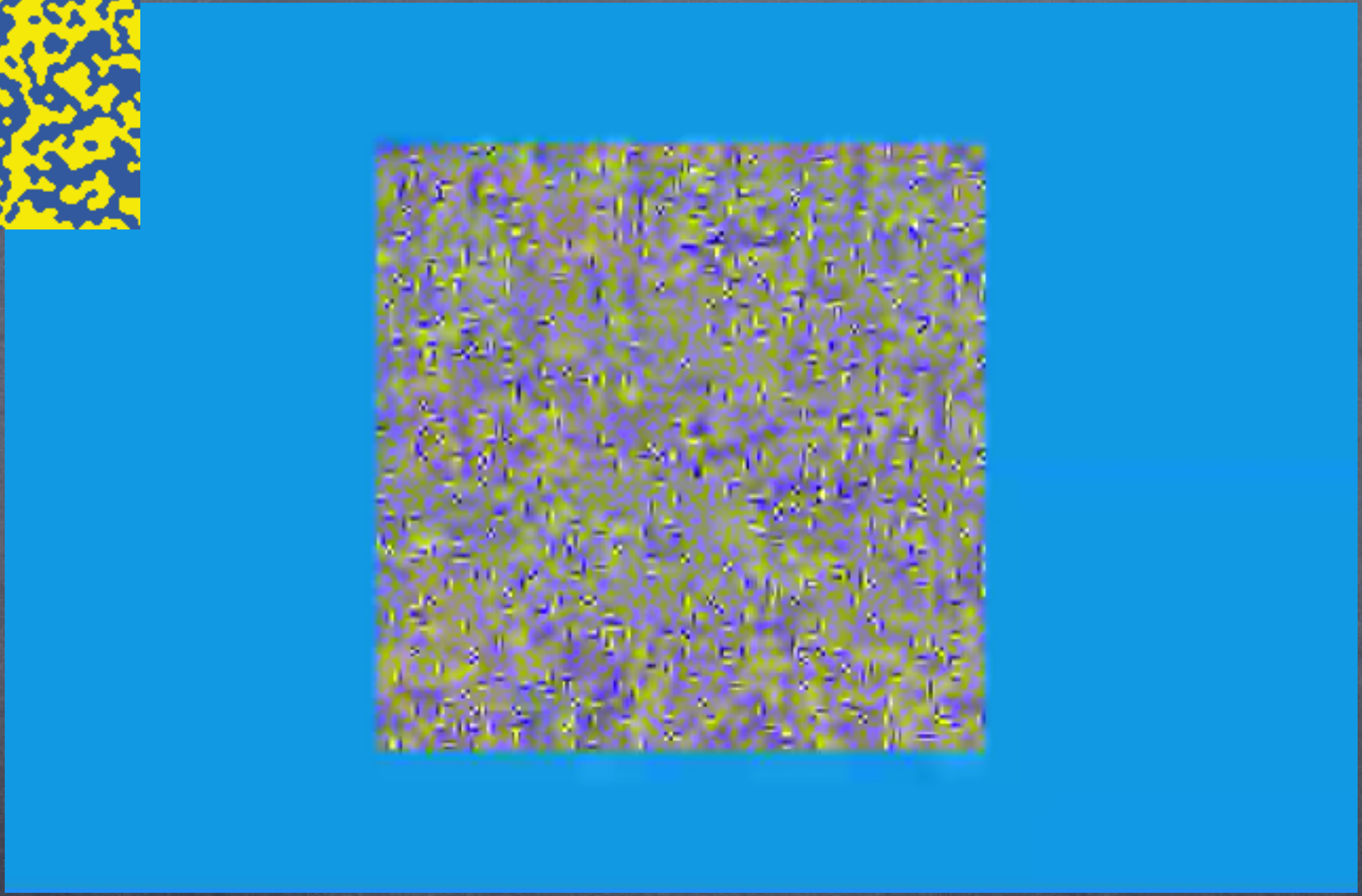
Length scale for $\theta = 5$



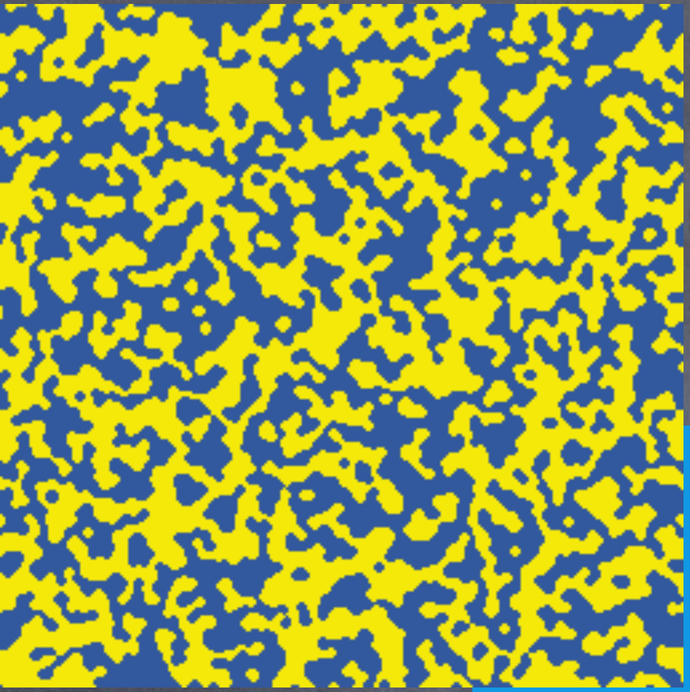
32 x 32 64 x 64 128 x 128 256 x 256 512 x 512



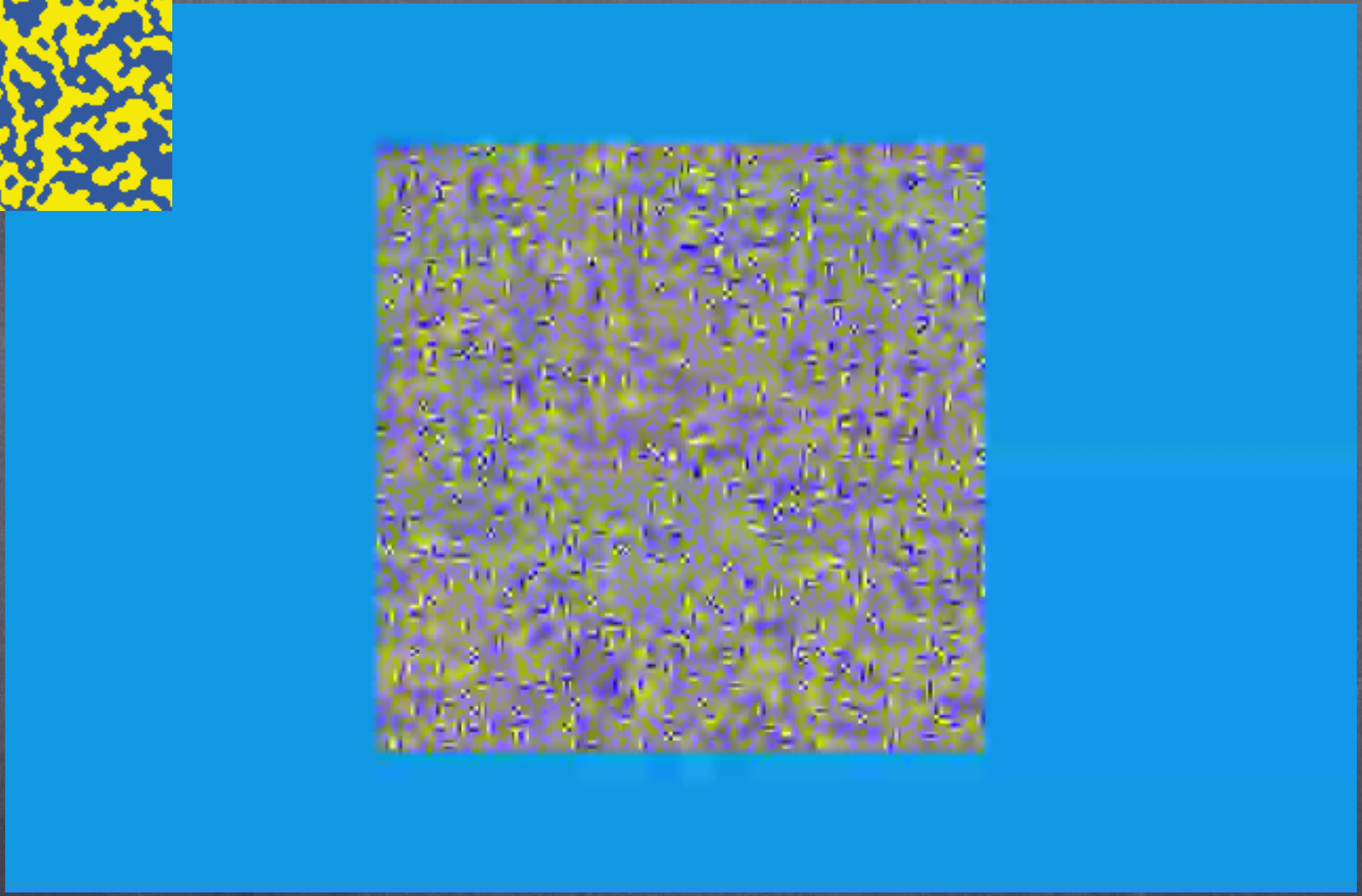
8 neighbors



Case of 44 neighbors

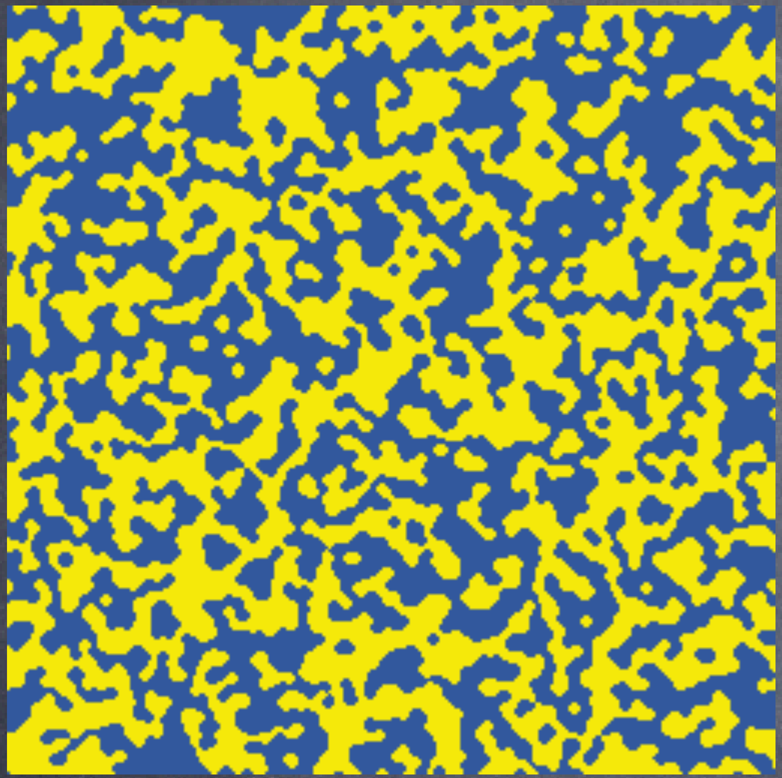


8 neighbors

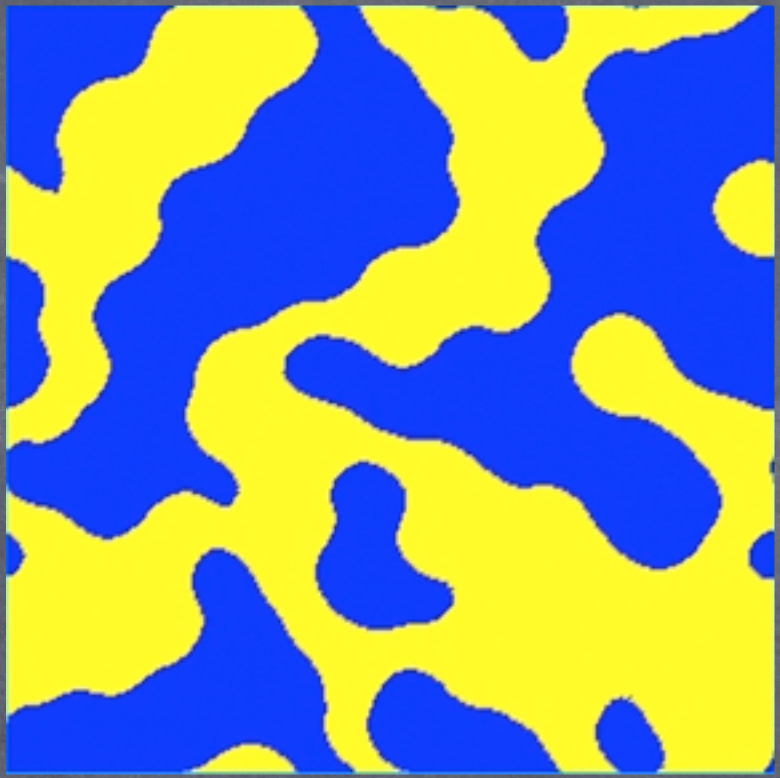


Case of 68 neighbors

Summary



8 neighbors



44 neighbors

68 neighbors

Quantitative behavior

For 8 neighbors and, if $\theta = 5$ and higher, then the "energy"

$$E[\{x\}] = -\frac{1}{2}\sum_{k=1}^{N} x_k \sum_{i \in V_k} x_i$$

decreases strictly during the evolution.

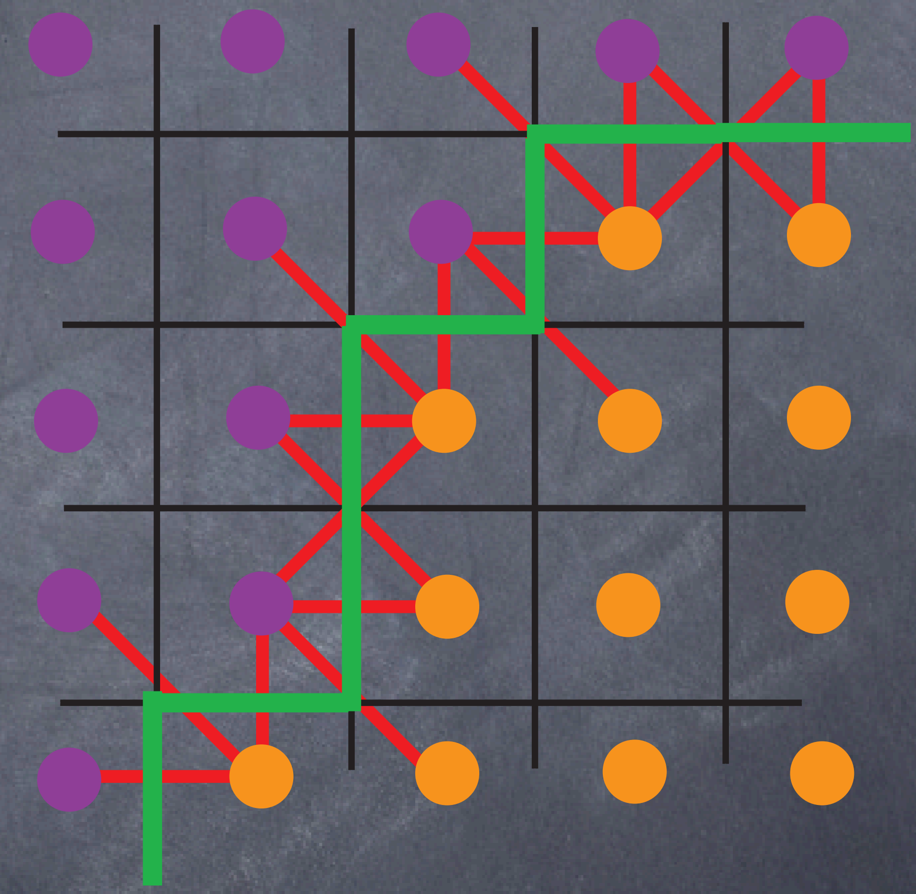
Moreover, $\Delta E_{kl} \leq 4\,(w_{kl} + 8 - 2\theta)$

where $w_{kl} \leq 1$

Geometrical interpretation

$$E[\{x\}] = -\frac{1}{2}\sum_{k=1}^{N}\sum_{i\in V_k} 1 + \frac{1}{2}\sum_{k=1}^{N}\sum_{i\in V_k}(1 - x_k x_i)$$

$$= -\frac{1}{2}8N + \frac{1}{2}\sum_{k=1}^{N}\sum_{i\in V_k}(1 - x_k x_i)$$



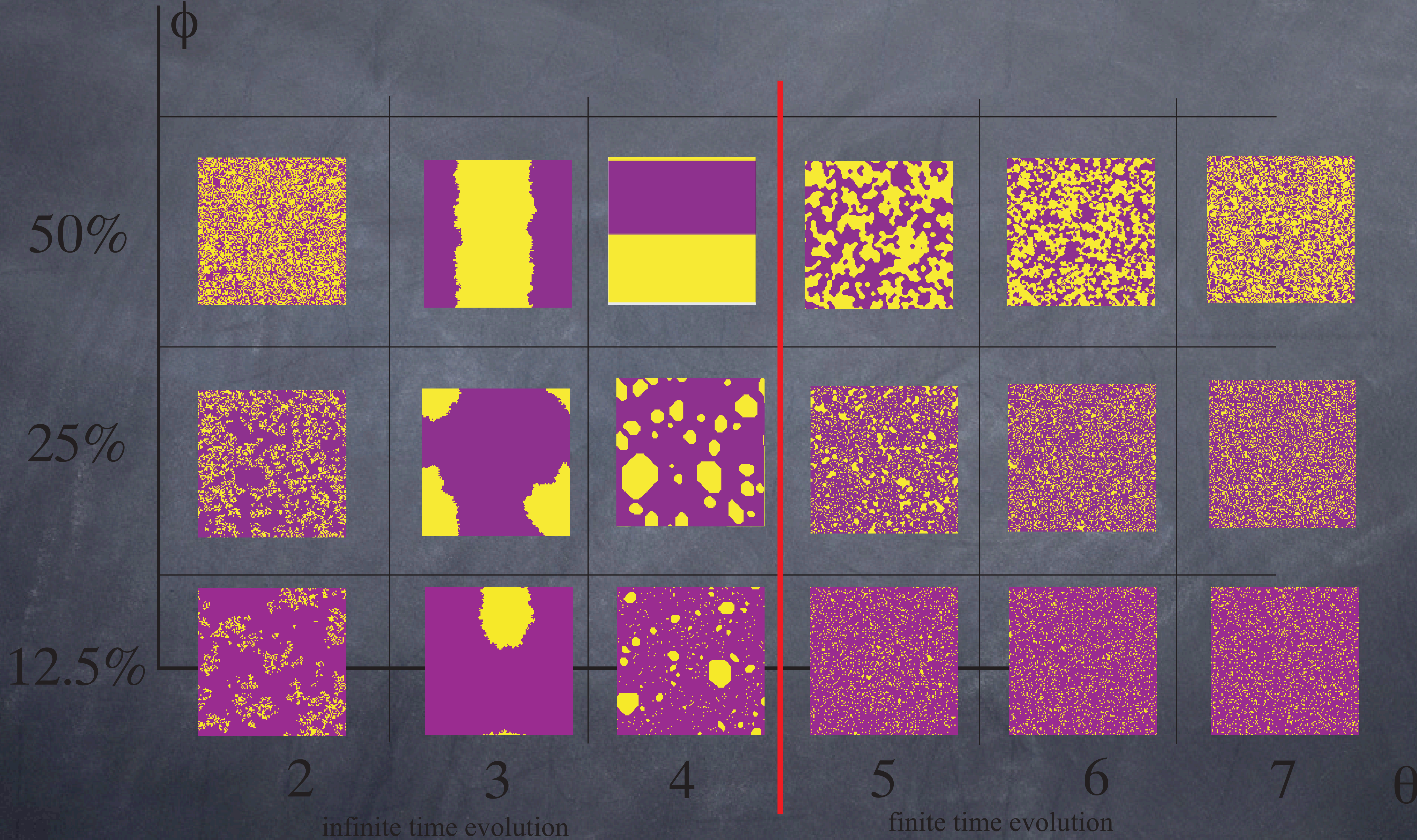
$$E = -4N + 2\times\left(3\sum \text{edges} - \sum \text{corners}\right)$$

$$= -4N + 2\times(3\times\text{perimeter} - \text{Nb. of corners}),$$

Few Consequences

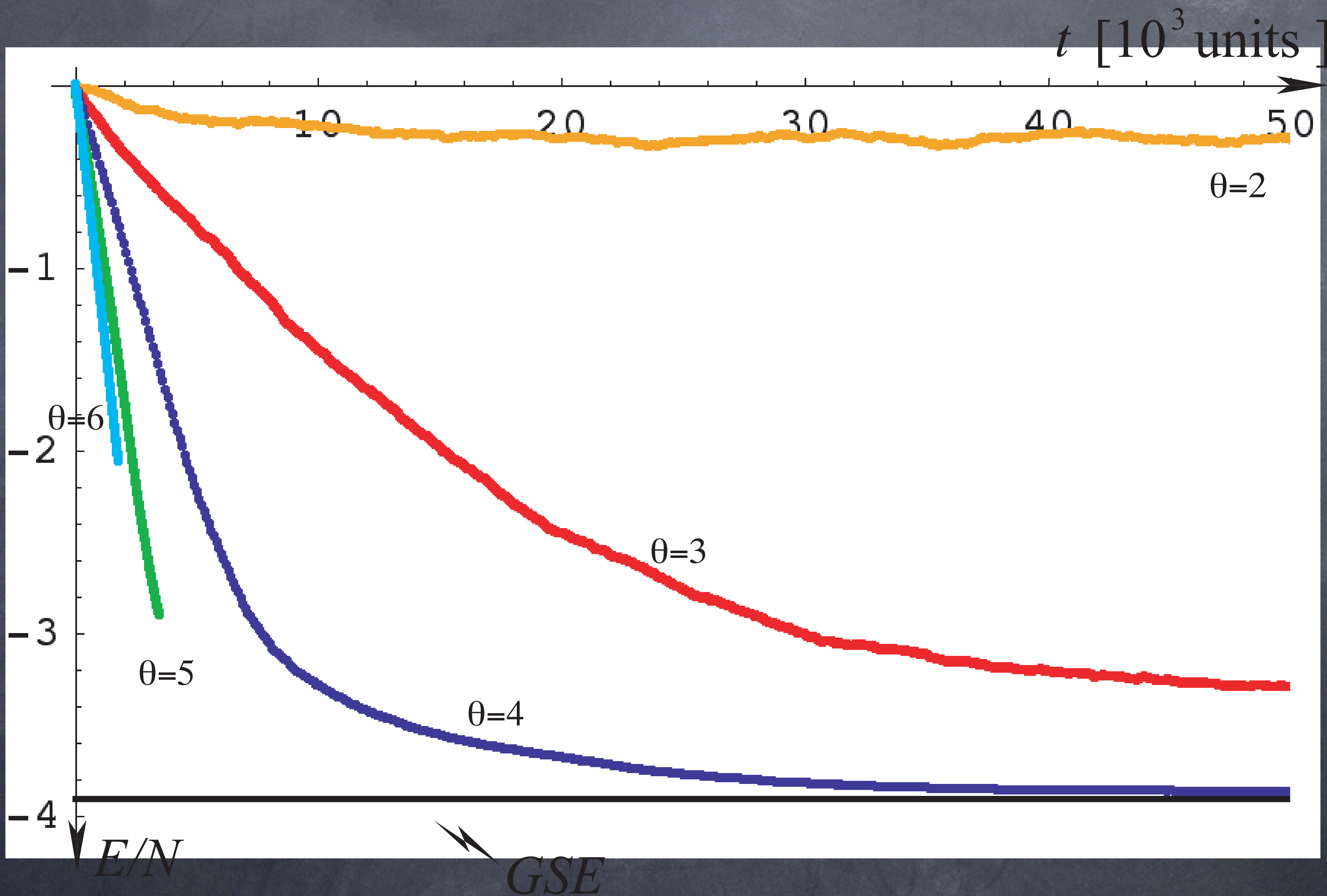
- Because the energy is bounded - 4 N ≤ E ≤ 4 N the dynamics is of finite time for $\theta = 5$ and higher.
- For $\theta \leq 3$ the dynamics continues indefinitely
- The case $\theta = 4$ may posses a complex dynamics
- The energy ground state.

Phase diagram

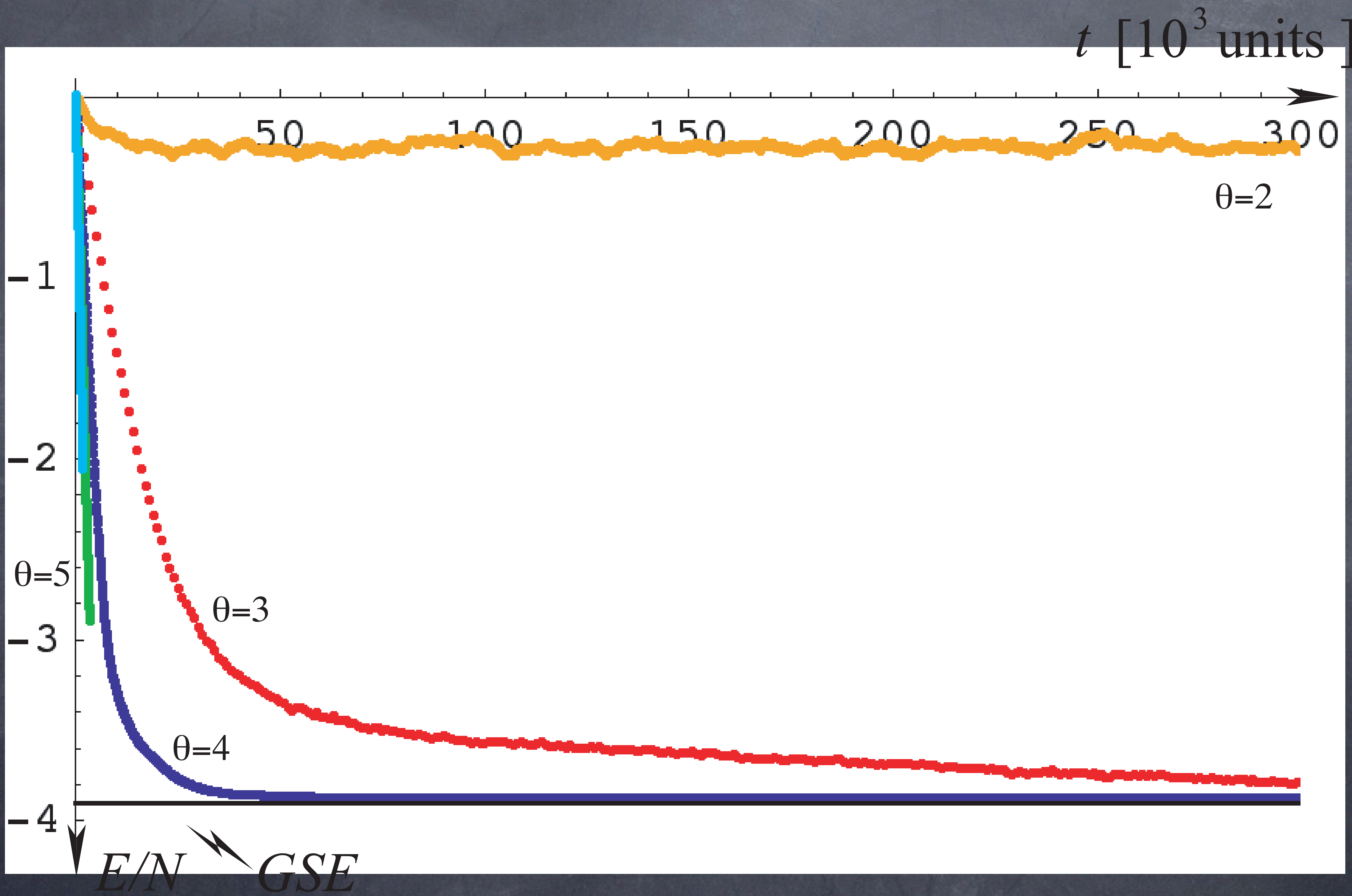


$$\phi = \frac{N_+}{N_+ + N_-}$$

E vs time



E vs time



Discussion

- Variants on the model and generalizations (graphs, non uniform tolerance, various states, protocols...)
- Q2R
- Segregation in higher dimensions ?

Segregation in 3D