EL UNIVERSO SEGÚN G
(On Goles Universal Machines)

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DISCO 2011, Valparaiso, Chile
For Eric's 60th Birthday

Foreword

Chip Firing Game

Neural Nets

Sand Piles

Ants

What do they share with ?

Turing machines

Cellular automata

Register machines

Boolean circuits

Universality

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Contents

Computability basics

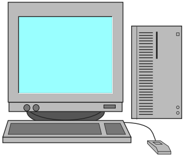
$\varphi_0, \varphi_1, \ldots$ *programming system*: listing which includes all the partial recursive functions of one argument over $\mathbb{N}$.

A programming system is *universal* if the partial function φ_{univ} s.t. $\varphi_{\text{univ}}(i, x) = \varphi_i(x)$ $\forall i, x \in \mathbb{N}$ is a p.r. function
(*ie.* if the system has a universal p.r. function)

Well known universal programming systems:

- Turing machines
- Cellular automata
- Circuits

Some terminology

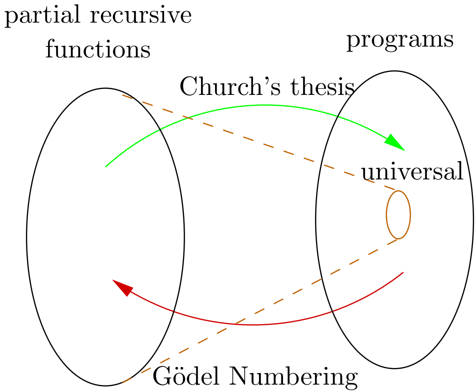


comput. theory	**“real world”**
programming system	computer
partial recursive function	program
φ_x	program x

Universality

Definition

A program is computation universal if it can compute any p.r. function.



Universality – More Details

Theorem

Given an indexing of the programs, there is a univ. p.r. function φ_{univ} s.t. if φ_x is the p.r. function computed by P_x, then, $\forall x, y$,

$$\varphi_x(y) = \varphi_{\mathit{univ}}(x, y)$$

φ_{univ} p.r. function $\Rightarrow$ there is a universal program.

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Goal of the talk

- Provide a “universality howto”
- Illustrations with constructions by Goles *et al.*

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Universality for Minsky

From Minsky's famous book (1967):

The universal machine as an interpretive computer
The universal machine will be given just the necessary materials: a description, on its tape, of T and of s_x (string of symbols corresponding to the entry); some working space; and the built-in capacity to interpret correctly the rules of operation as given in the description of T. Its behavior is very simple. U will simulate the behavior of T one step at a time...

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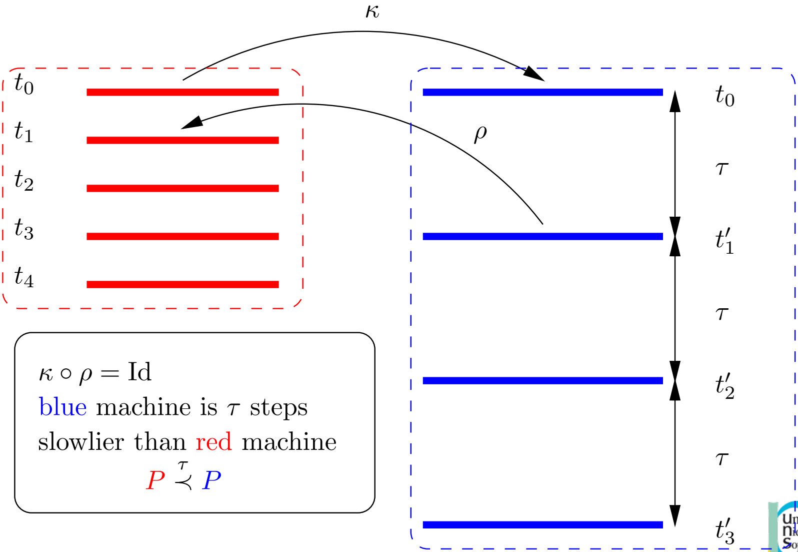
Simulations



$\kappa \circ \rho = \mathrm{Id}$

blue machine is τ steps slowlier than red machine

$$P \overset{\tau}{\prec} P$$

A classification

The M-machine P_U

- given the code i of any M'-machine and an input x can simulate $P'_i(x)$ — simulation universal
- simulates the behavior of a universal M'-machine — hereditary universal
- given an encoding $\chi(i)$ of a M'-machine, constructs a simulator of P'_i — construction universal

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The computational equivalence

Theorem

If there is an $\mathbf{M}$-program P_U which is either simulation-universal or hereditary-universal or construction-universal, then there is also an $\mathbf{M}$-computation-universal program.

- M has a simulation-universal program simulating any M'-program. M' has a computation-universal program. A M-program just has to simulate a computation-universal M'-program.
- From the computational point of view, simulation universality and hereditary universality coincide.
- by definition.

Universal Programs Howto

First choose a computer M. Two cases:

- from scratch (fixing the bootstap problem):
  - propose an indexing of the M-programs
  - build a M-program which can simulate any other M-program
- refer to an existing computer M'-with a universal program; construct either:
  - a M-program which simulates any M'-program
  - a M-program which simulates a universal M'-program.

Often, a chain of simulations is needed

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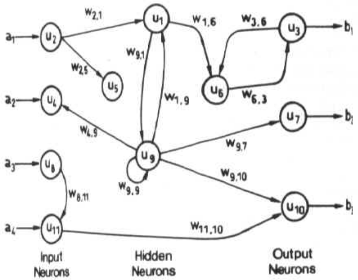
Neural networks



- binary states to the neurons x_i
- square matrix A
- binary vector b
- $x'_i = \mathbf{1}\left(\sum_{j \in V_i} a_{ij} x_j - b_i\right)$

Theorem (G., Matamala, 1997)

Any neural network $\mathcal{N}$ of size n can be simulated by a symmetric reaction-diffusion automaton with 3 states and of size $3(n+1)$.

Comments

- Change of the original graph structure (asymmetric)
- Careful weighting of the connecting edges
- Add a clock to the RDA

Universality comes from the universality of the Neural Networks (which is construction-universal).

Thus, 3-RDA are hereditary-universal.

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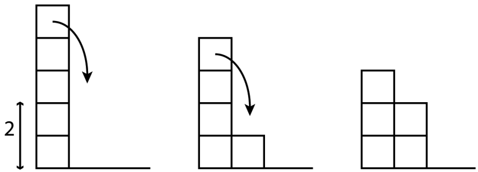
Sand Piles – Chip Firing Game



Theorem (G., Gajardo, 2005)

The sandpile over $\mathbb{Z}^2$ *with the von Neumann neighborhood of radius* $k \geq 2$ *is Turing universal.*

Using a graph connecting nodes: *Chip Firing Game*.

Theorem (G., Margenstern, 1997)

There is a universal parallel chip-firing game on an infinite connected undirected graph.

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Comments

For Sand Piles:

- Construct logical gates, wires + crossing information
- Sand pile logics follows Bank's construction connecting logic elements to create FSM used to simulate any Turing machine.

For Chip Firing Game:

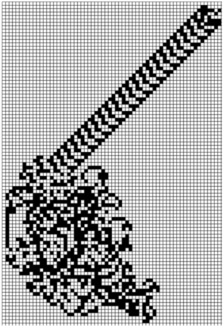
- Construction of logical gates, controller and registers
- Simulation of a two register machine (which simulates a universal Turing machine)

Both machines are hereditary universal

Ants

DDS moving an ant on a grid with states: {to-left, to-right}

- ant=arrow between 2 adjacent cells
- ant moves one cell forward at each time in the direction of its heading
- ant direction changes according to the cell where the ant arrives
- changes cell's state after the ant's visit



Single ant, all cells starting in to-left state, has a more or less symmetric trajectory in the first steps; then moves seemingly randomly until it starts building an infinite diagonal "highway".

Theorem (G., Moreira, Gajardo, 2002)

There is a universal single ant system over $\mathbb{Z}^2$.

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Comments

- Construction of logical gates and crossing information
- Ant's logics follows B.Durand's construction connecting logic elements to create FSM and uses them to simulate a CA.

Generalisation to $\Gamma(k, d)$ planar regular graphs of cardinality k and degree d as soon as $d = 3$ or 4.

Ant system is hereditary universal

Outline

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Conclusion

Simulation,hereditary and construction universality:

- General framework for constructing universal machines
- Allows P-completeness results, defines Chaitin complexity

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- provides hereditary universal machines
- chains of simulations
- underlying simulations are of various types:
  - 2-Register machines
  - Circuits
  - Cellular automata

with simple local interactions capable of complex global behaviors

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Thanks you for listening

Eric: Thanks for the results and... Bon anniversaire!