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A high order mixed-FEM for the Stokes problem on curved domains*

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Abstract

In this talk we propose and analize a high order mixed finite element method for a pseudostress-velocity formulation of the Stokes problem with Dirichlet boundary condition on curved domains. The method is based on approximating the fluid domain Ω by a polygonal/polyhedral subdomain D_h, where the Galerkin method is applied to approximate the solution, and on a transferring technique, based on integrating the extrapolated discrete gradient of the velocity, to approximate the Dirichlet boundary data on the boundary of D_h. Considering generic finite dimensional subspaces of $\mathrm{H}(\mathrm{div}, \mathrm{D}_h)$ for the pseudostress and of $\mathrm{L}^2(\mathrm{D}_h)$ for the velocity, we prove that the resulting Galerkin scheme becomes well-posed provided suitable hypotheses on the aforementioned subspaces are assumed. A feasible choice of discrete spaces is given by Raviart–Thomas elements of order $k \geq 0$ for the pseudostress and discontinuous polynomials of degree k for the velocity, yielding optimal convergence whenever the distance between both boundaries is of order h. Moreover, the pressure can be approximated optimally through a simple post-processing of the discrete pseudostress. We also derive an error analysis on the complement $\mathrm{D}_h^c := \Omega \backslash \overline{\mathrm{D}}_h$ for the extrapolated solutions. Finally, we provide some numerical experiments illustrating the good performance of the scheme and confirming the theoretical rates of convergence.

Key words: curved domain, high order, Stokes problem, mixed variational formulation

Mathematics subject classifications (2000): 65N30, 65N12, 65N15

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