S A N T I A G O N U M É R I C O III
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SANTIAGO, CHILE, JUNIO 28 - 30, 2017

A mixed virtual element method for a pseudostress-based formulation of linear elasticity *

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Abstract

In this talk we introduce and analyze a mixed virtual element method (mixed-VEM) for a pseudostress-displacement formulation of the linear elasticity problem with non-homogeneous Dirichlet boundary conditions. More precisely, we employ a mixed formulation which does not require symmetric tensor spaces in the finite element discretization. The main unknowns here are given by the pseudostress and the velocity, whereas physical quantities such as the stress, the strain tensor of small deformations, and the rotation, are computed through a simple postprocessing in terms of the pseudostress variable. We first recall the corresponding variational formulation, and then summarize the main mixed-VEM ingredients that are required for our discrete analysis. In particular, we utilize a well-known local projector onto a suitable polynomial subspace, in order to define a calculable version of our discrete bilinear form, whose continuous version requires information of the variables on the interior of each element. Next, we show that the global discrete bilinear form satisfies the hypotheses required by the Babuška-Brezzi theory. In this way, we conclude the well-posedness of our mixed-VEM scheme and derive the associated *a priori* error estimates for the virtual solution as well as for the fully computable projection of it. Furthermore, we also introduce a second element-by-element postprocessing formula for the pseudostress, which yields an optimally convergent approximation of this unknown with respect to the broken $\mathbb{H}(\mathbf{div})$-norm. In addition, this postprocessing formula can also be applied to the stress variable. Finally, several numerical results illustrating the good performance of the method and confirming the theoretical rates of convergence are presented.

Key words: linear elasticity, mixed virtual element method, a priori error analysis

Mathematics subject classifications (1991): 65N30, 65N12, 65N15, 76D07

*This work was partially supported by CONICYT-Chile through BASAL project CMM, Universidad de Chile; by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción; and by Universidad Nacional (Costa Rica), through the project 0106-16.

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