S A N T I A G O N U M É R I C O III
Noveno Encuentro de Análisis Numérico de Ecuaciones Diferenciales Parciales
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SANTIAGO, CHILE, JUNIO 28 - 30, 2017

A high-order singularity subtraction method for the Nyström discretization of boundary integral equations

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Abstract

We present a high-order singularity subtraction method for the Nyström discretization of Laplace and Helmholtz boundary integral operators and layer potentials. The proposed singularity subtraction approach allows integral operators and layer potentials to be expressed in terms of "smooth" integrands that can be easily and inexpensively evaluated by means of standard quadrature rules. The method relies on the use of Green's third identity and pointwise interpolation of the surface density in terms of homogeneous solutions of the associated PDE (harmonic polynomials in the case of the Laplace equation and plane-waves in the case of the Helmholtz equation). Used in conjunction with the Fast Fourier Transform, for evaluation of the surface derivatives of the density, and the Fast Multipole Method, for evaluation of non-local interactions, the proposed methodology enables second-kind integral equations to be solved in $O(N \log N)$ operations, where N denotes the number of discretization (quadrature) points on the boundary. A variety of numerical examples in two and three spatial dimensions—including smooth and piecewise smooth domains—demonstrate the capabilities of the proposed methodology and its advantages over high-order and spectrally accurate Nyström methods based on specialized quadrature rules.

Key words: Laplace equation, Helmholtz equation, Nyström method, singularity subtraction, Fast Multipole Method.

Mathematics subject classifications (2010): 45A05, 45E99, 30E25, 65R20

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