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Error analysis of an augmented mixed method for the Navier-Stokes problem with mixed boundary conditions*

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Abstract

In this paper we analyze an augmented mixed finite element method for the Navier-Stokes equations. More precisely, we extend the recent results from [2, Camaño *et al.*, Mathematics of Computation] to the case of mixed (Dirichlet and traction) boundary conditions in different parts of the boundary, and introduce and analyze a new pseudostress-velocity augmented mixed formulation for the fluid flow problem. The well-posedness analysis is carried out by means of a fixed-point strategy where the classical Babuška-Brezzi theory and the Banach's fixed-point Theorem are employed. Next, adapting to the discrete case the arguments of the continuous analysis, we are able to establish suitable hypotheses on the finite element subspaces ensuring that the associated Galerkin scheme becomes well-defined. Namely, a feasible choice of subspaces is given by Raviart-Thomas elements of order $k \geq 0$ and continuous piecewise polynomials of degree $k+1$ for the nonlinear pseudo-stress tensor and velocity, respectively, yielding optimal convergence rates. In addition, we derive a reliable and efficient residual-based a posteriori error estimator for our method. The proof of reliability exploits the global inf–sup condition and the local approximation properties of the Clément interpolant. On the other hand, the efficiency of the estimator follows from a combination of inverse inequalities and localization via edge–bubble functions. A set of numerical results is provided to exemplify the performance of the augmented method with mixed boundary conditions, to confirm the aforementioned properties of the a posteriori error estimator, and to show the behaviour of the associated adaptive algorithm.

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