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Polydisperse sedimentation in inclined channels*

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Abstract

In this talk we consider the flow of a fluid in a channel inclined by an angle θ. The fluid carries particulate matter consisting of small particles that belong to N different species differing in size and density. Here, the polydisperse transport and sedimentation process is modelled by combining a multilayer shallow water system with a polydisperse sedimentation model. The resulting model can be written as a hyperbolic system with nonconservative products of the form

$$\partial_t \vec{w} + \mathcal{A}(\vec{w})\partial_x \vec{w} + \mathcal{B}(\vec{w})\partial_y \vec{w} = G(\vec{w}) \tag{1}$$

(plus initial and boundary conditions), which we solve through finite volume techniques. The main difficulty is the definition of the nonconservative products that appear in the model. The unknowns of interest are the height h and the velocity field of the fluid $\vec{v}$ along with the concentrations by layer of the different solid species $\phi_{j,\alpha}$ for $j = 1, \ldots, N$ and $\alpha = 1, \ldots, M$. We show how to construct a high-order method to approximate the present hyperbolic system with nonconservative products, and report several numerical tests.

Key words: nonconservative products, finite volume method, hyperbolic systems

Mathematics subject classifications (1991): 65M06, 65M12, 76M25

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