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Unfitted finite element methods for PDEs on evolving surfaces*

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Abstract

Partial differential equations posed on evolving surfaces appear in a number of applications such as two-phase incompressible flows (surfactant transport on the interface) and flow and transport phenomena in biomembranes. Numerical approaches discussed in this report are based on Eulerian description of the surface PDE problem and employ a time-independent background mesh that is not fitted to the surface. The time-dependent surface $\Gamma(t) \subset \mathbb{R}^3$ is assumed smooth and closed for all $t \in [0, T]$. The evolution of the surface may be given implicitly, for example, by the level set method. As an example of the surface PDE we consider the transport–diffusion equation modelling the conservation of a scalar quantity u with a diffusive flux on $\Gamma(t)$, which is passively advected by a given smooth velocity field $\mathbf{w} : \mathbb{R}^3 \times [0, T] \to \mathbb{R}^3$,

$$\dot{u} + (\operatorname{div}_\Gamma \mathbf{w})u - \nu \Delta_\Gamma u = f \quad \text{on} \quad \Gamma(t), \quad t \in (0, T], \tag{1}$$

with initial condition $u(\mathbf{x}, 0) = u_0(\mathbf{x})$ for $\mathbf{x} \in \Gamma_0 := \Gamma(0)$. Here $\dot{u}$ denotes the advective material derivative, $\operatorname{div}_\Gamma$ is the surface divergence, Δ_Γ is the Laplace–Beltrami operator, and $\nu > 0$ is the constant diffusion coefficient. In the report, we discuss two unfitted finite element methods based on restrictions of outer (bulk, volumetric) finite element functions to the surface. This methodology is known as the trace finite element method (TraceFEM), see the recent review article [4]. In the first approach from [1, 2], one considers a weak formulation of (1) as a surface PDE on space–time manifold

$$\mathcal{S} = \bigcup_{t \in (0,T)} \Gamma(t) \times \{t\}, \quad \mathcal{S} \subset \mathbb{R}^4,$$

and a weak formulation of (1) as a surface PDE on $\mathcal{S}$. Further, one considers space–time prismatic elements in $\mathbb{R}^4$ and defines finite element counterparts of test and trial functions as traces of outer space–time polynomial finite element spaces on a tetrahedral reconstruction of $\mathcal{S}$. It turns out that the resulting method can be implemented in an efficient time-marching way. It has been also proved to be of the optimal first order (in

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space and time) in an energy norm and of the second order convergence in a weaker norm.Another unfitted finite element method we discuss is the one recently proposed in [3]. The main motivation for this method is to avoid space–time elements or any reconstruction of the space–time manifold. The method is based on trace of time-independent finite element spaces on a sequence od steady 2D surfaces and a hyperbolic solve (such as the Fast-Marching method) to find an extension of a function from a 2D surface to its 3D neighborhood.

Key words: surface PDEs, evolving surfaces, TraceFEM, level set method

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