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Solving unsteady coupled fluid mechanics and convective heat transfer problems by a geometric multigrid finite volume method*

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Abstract

Complexity in 2D fluid mechanics and natural convection problems arises from the coupling between continuity, linear momentum and energy equations caused by the buoyancy forces driven by density gradients being calculated through temperature differences. Increments on Grashof number increases the influence of the convective acceleration terms in the momentum equations, with velocity and temperature gradients being higher towards the external walls of the fluids container. Motion of the fluid is originated by temperature differences in the container walls. One of the difficulties in the solution of the natural convective problems using finite numerical methods with a single grid is that a high number of nodes is required to achieve accurate solution at high Rayleigh numbers. In such cases, the information provided by the boundary conditions is slowly communicated toward the center of the physical domain. Therefore the development of a novel geometric multigrid method, tested in the solution of three problems of increasing complexity via the finite volume method, is the objective of this paper. The cases studied in square cavities includes natural convection of air with $Ra = 10^3$ and 10^4; mixed convection of air inside a cavity with an inner solid either at the center or in the center of the right upper quarter section with Richardson numbers of 0.1 and 10 and solidification with natural convection of a binary alloy (Al-1.7%Si) with $Ra = 10^4$. The results of velocity profiles and streamlines are used to characterize the fluid mechanics while the temperature distribution allows the description of the convective heat transfer. Computing time required to solve each case with the geometric multigrid method, implemented with V, W and F restriction-prolongation cycles, is compared with the time used to achieve the solution with the same accuracy by using a single grid method. Savings of CPU time with the multigrid geometric method ranged from 12% for natural convection with solid-liquid phase up to 93% for natural convection.

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