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FEM approximation of sparse optimal control problems with finite–dimensional control space*

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Abstract

We consider the numerical approximation by the finite element method of a class of sparse optimal control problems governed by linear partial differential equations of elliptic type, which involves the ℓ_1–norm in the cost function. Optimal control problems with finite dimensional controls are motivated by many practical applications of optimal control problems with PDEs. Here, by technological requirements, controls are finite quantities which can be frequently identified with a vector in $\mathbb{R}^N$. Our research focuses of optimal control problems which entails the following main features:

- Controls are in a finite-dimensional control space
- Sparsity inducting term is considered in order to promote "simple solutions (those with many null entries)
- The state of the optimal control problem is a function where fnitely many pointwise state constraints on the state are imposed.

The problem of deriving error estimates has been addressed in [1] in the case of smooth cost functional, where the problem is translated in terms smooth nonlinear programming theory in finite dimensional and applying stability theory of generalized equations. Since in this case the cost function is non-smooth, the extension of this theory is not directly applicable. We bypass this difficulty by reformulating an alternative problem by exploiting the structure of the ℓ_1-norm, which allows to split the solution into its positive and negative parts. We are able to proof that for a parameter of discretization h, an order of convergence of $h|log(h)|$ is obtained. Numerical experiments are shown to underline our theory.

Key words: Finite element approximation, error estimates, sparse optimal control problems

Mathematics subject classifications (2010): 65M15, 49J20, 49J52

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